

An Effectiveness of Integrated Portfolio in Bancassurance

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1 Introduction

As is well known, the traditional banking business is to get finance via time deposit saving and make loans to firms. In that, a bank manages credit risk in its loan portfolio or equivalently credit risk pool to make a profit from the spread of interests between loan rate and deposit rate. On the other hand, in the insurance business, insurance risk is managed in its risk pool of the insured to make a profit from the difference between the premiums collected and the loss (damage) actually caused. There the separation or segregation between finance and insurance business is institutionally made on the basis that the function of making finance and the function of providing insurance against risks are regarded as different and separable at least from the side of the suppliers.

However, as the new terms bancassurance, assurebank and finassurance are sometimes used, a trend toward convergence of banking and insurance business is clearly observed in many financial commodities and business corporations. This trend will correspond to the needs of the firms and consumers who have to inseparably manage financial risk and insurance risk in their value development and life cycles. Merton (1994) and Kariya (2000) discussed the problem in view of functional approach to finance and insurance and provided clues to understanding the trend as natural .

Gora(1997) provided many actual examples of bancassurance and assurebank in Europe, Canada and USA. According to the author, bancassurance is the case where a bank sells insurance commodities developed by an insurance company the bank owns, and assurebanking is the reversed case. The author's basic viewpoint still lies in the institutional approach and he discussed about the merits and demerits of the business schemes of bancassurance and assurebanking.

In this paper, from a viewpoint of the traditional business schemes in banking and insurance, we consider the effectiveness of the convergence of banking business and insurance business. Specifically, regarding banking business and insurance business as the businesses which respectively make profits by pooling and managing the risks in their loan portfolio and policy portfolio, we consider the effectiveness of combining the

two portfolios or two businesses compared to each portfolio. The effectiveness is evaluated in terms of asymptotic default probability when each portfolio size is large. According to this criterion, the asymptotic default probability of an integrated portfolio is smaller than that of each portfolio under a certain condition.

The concept on which the insurance business relies for pooling and managing risk in its portfolio is a law of large numbers in probability theory. In a large and homogeneous pool of policies where the probability of the occurrence of an accident is constant, the frequency of accidents relative to the size of policies is regarded as close to constant. Hence so long as the rate of premium is set more than the probability of the occurrence, the probability that the collection of premiums received is bigger than the loss paid goes to one as the portfolio size gets large.

The principle of the law of large numbers is in fact used together with the diversification principle of risk in the management of a loan portfolio in the traditional banking. When the default probability of each loan is constants and the default events are independent, the default probability of a bank becomes smaller as the portfolio size gets larger, so long as a default-adjusted spread of loan interest rate and deposit interest rate is positive (Section 3). In this sense, risk management in banking business is similar to that in insurance business and the evaluation criterion based on the asymptotic default probability is reasonable.

This paper only treats one-period model for simplicity of arguments. A dynamic extension is left open for complication of modeling. Our results state that under a condition on default probability of loans, deposit rate, loan rate, premium rate, accident rate, and return rate for premiums, an integrated portfolio is more effective in asymptotic default probability than any of a bank portfolio and an insurance portfolio. Consequently the convergence of banking and insurance will be theoretically justified. In addition, the condition can be used to find a strategic position for a finansurance institution with loan rate, deposit rate and insurance premium as control variables.

The content of the paper is as follows.

Section 2 Principle of Traditional Insurance Business: Law of Large Numbers,

Section 3 Effect of Pooling Credit Risk and Insurance Risk on Portfolios,

Section 4 Effectiveness of a Finansurance Portfolio: General Case.

2 Principle of Traditional Insurance Business: Law of Large Numbers

As is discussed in Section 1, in the traditional insurance business a pool of insurance risks in a portfolio is managed relative to its reserve of premiums so that the business can be continued without default. Needless to say, the business scheme provides an

important function of risk management against insurable risk in our society. The theoretical concept on which the scheme of creating the function relies is the law of large numbers in probability theory. To develop our arguments in finansurance, let us briefly state the relevant point and introduce some notation we will use below.

Let the present time be denoted by 0 and consider one period model over [0,1]. Supposing a homogeneous insurance population for a specific risk, let n be the size of policies or equivalently the number of the objects (persons) to be insured in an insured portfolio and let \tilde{X}_i denote the claim amount when the i -th insured object gets an accident. Here we assume that each maximum claim for X_i is bounded by policy as $0 < X_i < A$. Then the total loss possibly caused in an insurance portfolio is expressed as

$$(2.1) \quad X = \tilde{X}_1 L_1 + \dots + \tilde{X}_n L_n ,$$

where L_i is the indicator function of the insurance event of the i th insured, defined by

$$(2.2) \quad L_i = \begin{cases} 1 & \text{if the } i\text{-th insured gets an accident ,} \\ 0 & \text{otherwise} \end{cases}$$

We call L_i the accident (event) generation function of the i th object. Here the losses $\tilde{X}_1, \dots, \tilde{X}_n$ are assumed to be independently and identically distributed (*i i d*) and the event generation functions L_1, \dots, L_n are also assumed to be *i i d*. It is often the case that $\{\tilde{X}_i\}$ and $\{L_i\}$ are also assumed. But here we do not assume it. Put

$$(2.3) \quad X_i = \tilde{X}_i L_i$$

and denote respectively the total loss and its mean loss per object by

$$(2.4) \quad X(n) = X_1 + \dots + X_n , \text{ and } \bar{X}(n) = X(n)/n .$$

Further assuming that the expectation of X_i is finite, let

$$\mu = E [X_i].$$

Then the weak law of large numbers states that for any $\epsilon > 0$

$$(2.5) \quad Q(|\bar{X}(n) - \mu| > \epsilon) \rightarrow 0 \quad (n \rightarrow \infty),$$

while the strong law of large numbers states

$$Q(\lim_{n \rightarrow \infty} \bar{X}(n) = \mu) = 1.$$

Here we use the weak law for simplicity in the sequel. Since by (2.5)

$$(2.6) \quad \begin{aligned} q(n, \mu + \varepsilon) &\equiv Q(\bar{X}(n) > \mu + \varepsilon) \\ &= Q(X(n) > n(\mu + \varepsilon)) \rightarrow 0 \quad (n \rightarrow \infty), \end{aligned}$$

so long as the premium is set as $\mu + \varepsilon$, the default probability goes to zero as $n \rightarrow \infty$ that the default event $\{\bar{X}(n) > \mu + \varepsilon\}$ of the insurance company occurs. But in reality, n is finite and hence the premium $\mu + \varepsilon$ is needed to be set to keep the default probability less than or equal to a certain level γ ;

$$(2.7) \quad q(n, \mu + \varepsilon) \leq \gamma.$$

Then, assuming that the variance σ^2 of X_i is finite, by the central limit theorem the default probability in (2.7) is approximately evaluated by the normal probability

$$(2.8) \quad q(n, \mu + \varepsilon) = 1 - \Phi\left(\sqrt{n} \frac{\varepsilon}{\sigma}\right) \leq \gamma,$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function. Let $\alpha(\gamma)$ be the γ -percentile point of the standard normal distribution. Then the equality in (2.8) holds if and only if

$$\sqrt{n} \varepsilon / \sigma = \alpha(\gamma).$$

Hence in order for (2.8) to be guaranteed, the insurance premium needs to be set at least more than or equal to

$$x_0 = \mu + \sigma \alpha(\gamma) / \sqrt{n},$$

Actually an insurance company usually regards the premium as

$$\text{Premium } x = \text{Pure Premium} + \text{Safety Loading} + \text{Expense}$$

In other words, the mean μ of X_i corresponds to the pure premium, $\sigma \alpha(\gamma) / \sqrt{n}$ to the safety loading, and the minimum premium to

$$x_0 = \mu(1 + \sigma \alpha(\gamma) / \mu \sqrt{n}) \equiv \mu(1 + \xi).$$

Then the expense is set as a proportion, say η , of the premium x , from which the premium is given by $x = x_0 + \eta x$ or equivalently

$$x = \mu(1 + \xi)/(1 - \eta).$$

The numerator of this formula is $\mu + \varepsilon$.

It is noted that the above argument holds even when \tilde{X}_i is constant, $\tilde{X}_i \equiv A$, in $X_i = \tilde{X}_i L_i$. This is because L_i is random and the central limit theorem holds for the sum of L_i 's.

3 Effect of Pooling Credit Risk and Insurance Risk on Portfolio

To treat a central part of our problem, we first consider the following simplified one period insurance problem. Though it is deleted later, we assume that when an accident occurs in the period, a fixed amount A is paid without respect to the size of its damage or loss. This assumption corresponds to the assumption of 100% damageability in insurance. Then in the above setting the problem of controlling the probability that an insurance portfolio (or company) gets defaulted against a homogeneous insurance population is regarded as the problem of controlling the probability that

$$(3.1) \quad U_2 = AY_2 - n_2x(1 + r_2)A$$

is positive. Here x is a premium, r_2 is a rate of interest for the premium reserve, $n_2 \equiv n$, $L_{2i} \equiv L_i$ and

$$(3.2) \quad Y_2 = L_{21} + \dots + L_{2n_2}$$

is the total number of accidents possibly occurring in one period for the portfolio. The first term AY_2 of the right side in (3.1) is the total loss amount the insurance company has to pay at time 1. The second term is the premium revenue n_2xA received at time 0 plus interest received at time 1 under the rate r_2 . Therefore $U_2 > 0$ means default for the insurance portfolio.

Now to consider a convergence problem of finance and insurance, we first define the default of a loan portfolio in banking for a homogeneous credit risk population where each loan size is assumed to be A . Corresponding to the default problem in insurance, the default of the loan portfolio is defined as the situation that

$$(3.3) \quad \begin{aligned} U_1 &= n_1A(1 + r^*) - A(1 + r_1)(n_1 - Y_1) \\ &= A(1 + r_1)Y_1 - n_1A(r_1 - r^*) \end{aligned}$$

is positive, where r_1 is a loan interest rate, r^* is a saving rate and

$$(3.4) \quad Y_1 = L_{11} + L_{12} + \dots + L_{1n_1}$$

is the total number of defaulted loans in the total size n_1 of the loan portfolio. Here it is assumed that the recovery rate of a defaulted loan is zero and the default generation functions L_{1j} are independent. The first term of the right side of (3.3) in the first equality is the amount the bank has to pay back with interest at time 1 and the second term is the remaining amount when Y_2 firms get bankrupted during the period. The second equality of (3.3) means the difference between the total loss and the revenue from the interest spread $r_1 - r^*$. Hence the default problem is regarded as the problem of controlling the probability that $U_1 > 0$.

In the argument below we view the convergence problem as the problem of finding a condition for an effectiveness of the combined portfolio of the insurance risk portfolio and the credit risk portfolio. To treat this problem, first note that Y_j 's are independently distributed as binomial;

$$(3.5) \quad Y_j \sim B_i(n_j, p_j) \quad p_j = E[L_{ji}] \quad (j=1,2),$$

where p_j is the default probability of individual firm ($j=1$) or insured object ($j=2$) in one period. Hence as is well known, they are approximated as normal;

$$(3.6) \quad Z_j \equiv (Y_j - n_j p_j) / (n_j \sigma_{jj})^{1/2} \sim N(0,1) \quad \text{with} \\ \sigma_{jj} = p_j(1 - p_j).$$

Using this fact, we evaluate the default probabilities of individual portfolios and the integrated portfolio approximately.

(1) Default probability of a bank portfolio

Expressing U_1 in terms of Z_1 in (3.4) as

$$(3.7) \quad U_1 = \sqrt{n_1 + n_2} A \left\{ (1 + r_1) \sqrt{m_1 \sigma_{11}} Z_1 - \sqrt{n_1 + n_2} m_1 [(r_1 - r^*) - p_1(1 + r_1)] \right\}$$

the default event $U_1 > 0$ of a loan portfolio is equivalent to the event

$$Z_1 > \sqrt{n_1 + n_2} a_1, \quad \text{where}$$

$$\begin{aligned}
(3.8) \quad & a_1 = \sqrt{m_1} u_1 / \sqrt{V_{11}} \\
& m_1 = n_1 / (n_1 + n_2), \quad v_{11} = (1+r)^2 \sigma_{11} \quad \text{and} \\
& u_1 = (r_1 - r^*) - p_1(1+r_1).
\end{aligned}$$

Here u_1 is the spread of loan and saving rates when the default probability p_1 of an individual firm is taken into account, which we shall call the default-adjusted spread or simply adjusted bank spread. Here $u_1 > 0$ is equivalent to $(r_1 - r^*) / (1+r_1) > p_1$, which is satisfied when p_1 is small. We assume $u_1 > 0$. Hence when n_1 is large, the default probability of a bank portfolio is approximately evaluated as

$$(3.9) \quad Q(U_1 > 0) \approx 1 - \Phi(\sqrt{n_1 + n_2} a_1).$$

It is easy to see that when u_1 is negative, a_1 is negative and so the default probability is larger than 1/2. This implies that a bank cannot form a loan portfolio unless the discounted spread of loan and saving rates is bigger than the default rate p_1 .

(2) Default of an insurance portfolio

The default event $U_2 > 0$ of an insurance portfolio is equivalent to the event $Z_2 > \sqrt{n_1 + n_2} a_2$ since U_2 is expressed as

$$(3.10) \quad U_2 = \sqrt{n_1 + n_2} A \left\{ \sqrt{m_2 \sigma_{22}} Z_2 - \sqrt{n_1 + n_2} m_2 ((1+r_2)x - p_2) \right\},$$

where

$$\begin{aligned}
(3.11) \quad & a_2 = \sqrt{m_2} u_2 / \sqrt{\sigma_{22}}, \\
& m_2 = n_2 / (n_1 + n_2), \quad \text{and} \\
& u_2 = (1+r_2)x - p_2.
\end{aligned}$$

Here u_2 is the difference between the insurance premium $(1+r_2)x$ viewed at time 1 and the default rate p_2 of an insured object, which we shall call the insurance spread. Again when $u_2 \leq 0$, the default probability of an insurance portfolio becomes greater than or equal to 1/2. Hence we assume $u_2 > 0$. For a large n_2 , the default probability is expressed as

$$(3.12) \quad Q(U_2 > 0) \approx 1 - \Phi(\sqrt{n_1 + n_2} a_2).$$

(3) Default of the integrated portfolio

In the integrated portfolio of a loan portfolio and an insurance portfolio, the default event becomes $U_1 + U_2 > 0$. For notation, setting $v_{22} \equiv \sigma_{22}$, $U_1 + U_2$ is expressed in terms of Z_1 and Z_2 as

$$(3.13) \quad U_1 + U_2 = \sqrt{n_1 + n_2} A \left\{ \sqrt{m_1 v_{11}} Z_1 + \sqrt{m_2 v_{22}} Z_2 - \sqrt{n_1 + n_2} [m_1 u_1 + m_2 u_2] \right\},$$

where v_{11} is given in (3.6). Here letting

$$(3.14) \quad W = [\sqrt{m_1 v_{11}} Z_1 + \sqrt{m_2 v_{22}} Z_2] / (m_1 v_{11} + m_2 v_{22})^{1/2},$$

$U_1 + U_2 > 0$ is equivalent to

$$W > \sqrt{n_1 + n_2} b,$$

where

$$(3.15) \quad b = (m_1 u_1 + m_2 u_2) / (m_1 v_{11} + m_2 v_{22})^{1/2}.$$

Therefore when n_1 and n_2 are large and satisfy

$$(3.16) \quad m_i \rightarrow \delta_i \text{ (constant)} \quad (\min(n_1, n_2) \rightarrow \infty),$$

the default probability of the integrated portfolio is approximated as

$$(3.17) \quad Q(U_1 + U_2 > 0) \approx 1 - \Phi(\sqrt{n_1 + n_2} b).$$

Now let us consider a condition for which the integrated portfolio is more effective than any of the bank portfolio and the insurance portfolio in terms of default probability. Clearly from (3.9) (3.12) and (3.17) , a condition is

$$(3.18) \quad b > a_i \quad (i = 1, 2).$$

The condition $b > a_1$ is expressed as

$$(3.19) \quad z > z_1 \equiv \frac{1}{\beta_1} - 1,$$

where

$$(3.20) \quad z = m_2 u_2 / m_1 u_1,$$

$$\beta_i = [m_i v_{ii} / (m_1 v_{11} + m_2 v_{22})]^{1/2}. \quad (i = 1, 2)$$

Also the condition $b > a_2$ is expressed as

$$(3.21) \quad z_2 \equiv \beta_2 / (1 - \beta_2) > z.$$

Note that $z_2 > z_1$ is always satisfied.

Theorem 3.1 Assume (3.16). Then a necessary and sufficient condition for the integrated portfolio to be asymptotically more effective than any of the portfolios is $z_2 > m_2 u_2 / m_1 u_1 > z_1$ or equivalently

$$(3.22) \quad 1 - \left(\frac{m_2 \sigma_{22}}{m_1 (1+r_1)^2 \sigma_{11} + m_2 \sigma_{22}} \right)^{1/2} < \frac{m_1 u_1}{m_1 u_1 + m_2 u_2} < \left(\frac{m_1 (1+r_1)^2 \sigma_{11}}{m_1 (1+r_1)^2 \sigma_{11} + m_2 \sigma_{22}} \right)^{1/2},$$

where

$$u_1 = (r_1 - r^*) - p_1(1+r_1), \quad u_2 = (1+r_2)x - p_2, \quad \text{and} \quad m_i = n_i / (n_1 + n_2)$$

are respectively the bank spread, the insurance spread and the proportion of the individual portfolio sizes in the integrated portfolio.

The condition is of course a condition on the set

$$(r_1, r_2, p_1, p_2, r^*, x, m_1, m_2).$$

Hence when the portfolio sizes n_1 and n_2 and default rates p_1 in banking and p_1 in insurance are given, this condition gives a restriction on the set $\{r_1, r_2, r^*, x\}$, each of which is the component of the prices {loan rate, return rate for premiums, insurance premium} of the finanssurance business. A strategic choice of a point in this set will give a finanssurance institution a better portfolio position.

When $n_1 = n_2 = n$, the condition gives no restriction on n , while $p_1 = p_2 = p$, it gives a linear restriction on p .

As a special case, let us consider the case where the portfolio sizes are equal ($n_1 = n_2$) and the individual default rates are equal ($p_1 = p_2 = p$). Then we have

Corollary Suppose $n_1 = n_2$, $p_1 = p_2 = p$. Then the condition (3.22) is equivalent to the condition of (3.23) and (3.24);

$$(3.23) \quad \begin{aligned} (r_1 - r^*)[\sqrt{(1+r_1)^2 + 1} - (1+r_1)] / (1+r_1)(1+r_2) &< x \\ &< (r_1 - r^*)[\sqrt{(1+r_1)^2 + 1} + 1] / (1+r_1)(1+r_2)^2 \end{aligned}$$

and

$$(3.24) \quad a_1 < p < a_2 ,$$

where

$$a_1 = \frac{\beta_1[(r_1 - r^*) + (1 + r_2)x] - (r_1 - r^*)}{\beta_1(2 + r_1) - (1 + r_1)},$$

$$a_2 = \frac{\beta_2[(r_1 - r^*) + (1 + r_2)x] - (1 + r_2)x}{\beta_2(1 + r_1) - (1 - \beta_2)}.$$

Here

$$\beta_2^2 = 1 / [(1 + r_1)^2 + 1] = 1 - \beta_1^2.$$

Proof. The condition $z > m_2 u_2 / m_1 u_1$ is equivalent to

$$(3.25) \quad \beta_2(r_1 - r^*) - (1 + r_2)x(1 - \beta_2) > p[\beta_2(1 + r_1) - (1 - \beta_2)].$$

Here since the right side is easily shown to be positive, the left side need to be positive,

which gives

$$(3.26) \quad 0 < x < (r_1 - r^*)[1 + \sqrt{1 + (1 + r_1)^2}] / (1 + r_1)^2(1 + r_2).$$

This is the right inequality of (3.21). On the other hand, from $m_2 u_2 / m_1 u_1 > z_1$, we obtain

$$(3.27) \quad p[(1 + r_1)(1 - \beta_1) - \beta_1] > (r_1 - r^*)(1 - \beta_1) - \beta_1(1 + r_2)x.$$

Here since the inside of [] is negative, the right side must be negative, which leads to the left inequality of (3.23).

Therefore when (3.23) is satisfied, combining (3.25) and (3.27) yields the result in (3.24).

Now we delete the assumption of 100% damageability. In the case of non-life (damage and casualty) insurance, when an accident occurs, the full payment of the insurance is not paid, but only the loss caused by the damage is paid with a maximum level A . To treat this general situation, let

$$(3.28) \quad w_i = X_i / A.$$

Also, in the case of loan, when a firm gets bankrupted, some portion of the loan made to the firm is recovered. Hence let the loss rate be w_i , or equivalently let $\tilde{X}_i = w_i A$

in (2.3). Then in either case, w_i is a random variable on $[0, 1]$ and U_1 and U_2 are expressed as

$$(3.29) \quad \begin{aligned} U_1 &= A(1+r_1) \sum_{j=1}^{n_1} w_{1j} L_{1j} - n_1 A(r_1 - r^*), \quad \text{and} \\ U_2 &= A \sum_{j=1}^{n_2} w_{2j} L_{2j} - n_2 x(1+r_2) A. \end{aligned}$$

Under the assumption of the independence of $\{L_{ij}\}$ and $\{w_{ij}\}$

$$(3.30) \quad \begin{aligned} E[w_{ij} L_{ij}] &= \xi_i p_i \equiv \mu_i, \quad \text{with} \quad E[w_{ij}] = \xi_i, \\ \text{Var}(w_{ij} L_{ij}) &\equiv \sigma_{ii} = E(w_{ij}^2) p_i (1 - p_i). \end{aligned}$$

and the standardized variate

$$(3.31) \quad Z_i \equiv \left[\sum_{j=1}^{n_i} w_{ij} L_{ij} - n_i \mu_i \right] / (n_i \sigma_{ii})^{1/2}$$

is asymptotically standard normal. Consequently replacing p_1 by μ_1 in (3.7) and (3.8), p_2 by μ_2 in (3.10) and (3.12), σ_{ii} in (3.6) by σ_{ii} in (3.30), and the spreads u_1 and u_2 in (3.8) and (3.11) by

$$(3.32) \quad \begin{aligned} u_1 &= (r_1 - r^*) - p_1 \xi_1 (1+r_1) \quad \text{and} \\ u_2 &= (1+r_2)x - p_2 \xi_2, \end{aligned}$$

respectively the argument made above holds as it stands.

Theorem 3.2. In a general case with a common level A for loan and insurance, the condition (3.22) is necessary and sufficient for an integrated portfolio to be more effective, where σ_{ii} and u_i are defined by (3.30) and (3.32)

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4 Effectiveness of a Fininsurance Portfolio :General Case

In this section, let us consider a general case where there are K_1 homogeneous credit risk populations with different individual default rate p_{1k} 's for each population and different loan sizes A_k 's ($k = 1, \dots, K_1$) and there are K_2 homogeneous insurance risk

populations with different default rates p_{2k} for each population and different insurance sizes ($k = 1, \dots, K_2$).

The default problem in banking is the problem of controlling the default probability of a bank portfolio with loans of different credit ratings. This problem is treated in the same way as in Section 3. Here we only treat the case where the recovery rate is zero, i.e., $w_{ij} = 1$ in (3.28), since the nonzero case is a direct extension of the zero case, as has been seen in Section 3. Then the default probability is the probability that the event $U_1 > 0$ occurs where

$$(4.1) \quad U_1 = \sum_{k=1}^{K_1} A_k \left\{ (1 + r_{1k}) Y_{1k} - (r_1 - r^*) n_{1k} \right\}, \quad \text{with}$$

$$Y_{1k} = L_{11}(k) + \dots + L_{1n1k}(k).$$

Here k denotes the k -th credit rating ($k = 1, \dots, K_1$), r_{1k} is the loan rate for firms with the in k th rating, n_{1k} is its number of the loans, and A_k is the loan size. Of course, the loan size is not common to all the firms of the k -th rating in general. But the argument below is easily extended with some complication in notation to a general case where there are M_k classes in loan size for each rating k , as in Credit Risk Plus(1997). Hence without loss of generality, we take this simplified setting.

Rewrite U_1 in (4.1) as

$$(4.2) \quad U_1 = A \sum_{k=1}^{K_1} a_k \left\{ (1 + r_{1k}) \sqrt{n_{1k} \sigma_{1kk}} Z_{1k} - n_{1k} u_{1k} \right\} \quad \text{with}$$

$$u_{1k} = (r_1 - r^*) - (1 + r_{1k}) p_{1k},$$

$$A = \sum_{k=1}^{K_1} A_k, \quad a_k = A_k / A, \quad \text{and} \quad \sigma_{1kk} = p_{1k} (1 - p_{1k}).$$

Also assume that

$$\text{as } \min_k n_{1k} \rightarrow \infty, \quad n_{1k} / \sum_j n_{1j} \rightarrow \delta_{1j} > 0.$$

Then binomial variates $\{Y_{1k}\}$ are asymptotically multivariate normal in the sense that $\{Z_k\}$ asymptotically follows

$$(4.3) \quad Z(1) = (Z_1, \dots, Z_{K_1}) \sim N(0, \Lambda(1)) \quad \text{with} \quad \Lambda(1) = (\rho_{1ij}),$$

where ρ_{1ij} is the correlation of Z_{1i} and Z_{1j} . Consequently

$$U_1 \sim N(\mu_1, \gamma_{11}) \quad \text{with}$$

$$\begin{aligned}
(4.4) \quad \mu_1 &= A \sum_{k=1}^{K_1} a_k n_{1k} u_{1k}, \\
\gamma_{11} &= A^2 \sum_i \sum_j a_i a_j (1+r_{1i})(1+r_{1j}) \sqrt{n_{1i} n_{1j}} \sigma_{1ij}, \quad \text{and} \\
\sigma_{1ij} &= (\sigma_{1ii} \sigma_{1jj})^{1/2} \rho_{1ij}.
\end{aligned}$$

On the other hand, also in the case of an insurance portfolio with different insurance risks, the default probability is described as the probability that $U_2 > 0$, where

$$\begin{aligned}
(4.5) \quad U_2 &= \sum_{k=1}^{K_2} B_k \{Y_{2k} - x_k(1+r_2)n_{2k}\}, \quad \text{and} \\
Y_{2k} &= L_{21}(k) + \dots + L_{2n_{2k}}(k).
\end{aligned}$$

Here the return rate of managing the reserve of premiums is assumed to be r_2 .

Then similarly to (4.2), rewrite U_2 as

$$\begin{aligned}
(4.6) \quad U_2 &= B \sum_{k=1}^{K_2} b_k \left\{ \sqrt{n_{2k} \sigma_{2kk}} Z_{2k} - n_{2k} u_{2k} \right\} \quad \text{with} \\
u_{2k} &= (1+r_2)x_k - p_{2k}, \\
B &= \sum_{k=1}^{K_2} B_k, \quad b_k = B_k/B, \quad \text{and} \\
\sigma_{2kk} &= p_{2k}(1-p_{2k}).
\end{aligned}$$

Then $\{Z_{2k}\}$'s are asymptotically multivariate normal;

$$(4.7) \quad Z(2) = (Z_{21}, \dots, Z_{2K_2}) \sim N(0, \Lambda(2)) \quad \text{with} \quad \Lambda(2) = (\rho_{2ij}).$$

Here it is also assumed that

$$\text{as } \min n_{2k} \rightarrow \infty, \quad n_{2k} / \sum n_{2j} \rightarrow \delta_{2k} > 0.$$

Consequently

$$\begin{aligned}
(4.8) \quad U_2 &\sim N(\mu_2, \gamma_{22}) \quad \text{with} \\
\mu_2 &= B \sum_{k=1}^{K_2} b_k n_{2k} u_{2k}, \\
\gamma_{22} &= B^2 \sum_i \sum_j b_i b_j \sqrt{n_{2i} n_{2j}} \sigma_{2ij} \quad \text{and} \\
\sigma_{2ij} &= (\sigma_{2ii} \sigma_{2jj})^{1/2} \rho_{2ij}.
\end{aligned}$$

Therefore the default problem of the integrated portfolio of credit risk and insurance risk is the problem of controlling the probability that $U_1 + U_2 > 0$. Here we assume

that as $\min_k (\min_k n_{1k}, \min_j n_{2j}) \rightarrow \infty$

$$(4.9) \quad \sum n_{1k} / (\sum n_{1k} + \sum n_{2k}) \rightarrow \xi_1.$$

Theorem 4.1 For the integrated portfolio to be asymptotically more effective than any of the bank portfolio and the insurance portfolio, it is necessary and sufficient that

$$(4.10) \quad \frac{\beta_2}{1-\beta_2} < \frac{\mu_2}{\mu_1} < \frac{1}{\beta_1} - 1, \quad \text{where}$$

$$\beta_i = [\gamma_{ii} / (\gamma_{11} + \gamma_{22})]^{1/2} \quad (i=1,2).$$

The proof is the same as the case in Section 3. Note that (4.10) is equivalent to

$$(\mu_1 + \mu_2) / \sqrt{\gamma_{11} + \gamma_{22}} > \mu_i / \sqrt{\gamma_{ii}} \quad (i=1,2).$$

Of course in Theorem 4.1 the independence of credit risk and insurance risk is assumed.

5 Conclusion

This paper specifically gives an explicit condition for which an integrated portfolio or equivalently finansurance portfolio is better in asymptotic default probability than any individual portfolio: bank portfolio and insurance portfolio. It can be used to find a strategic position for a finansurance institution and to make a complex commodity with credit risk and insurance risk.

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