Education inequality among different social groups*

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August 31, 2010

Abstract

In this paper, we study an education planning problem Agents differ in talents to acquire education and in social group memberships (such as race). They exert costly effort to acquire education. Income and public help for education are distributed among the agents to maximize a social welfare function. We compare two allocations derived from the *Rawlsian* (or *maximin*) and the *utilitarian* social welfare functions. Our main findings are as follows: Education policies derived from the Rawlsian social welfare function induce a form of *reverse discrimination* in the sense that a more disadvantaged socal group achieves higher education level. In contrast, policies derived from the utilitarian social welfare function do not induce such reverse discrimination.

1 Introduction

Inequality of education among individuals is often considered as a major cause of social inequalities among them. Especially, social inequalities among different *social groups* such as genders and races would be, at least partially,

*The author is grateful to Mayu Atoda for stimulating discussions. The author thanks Eiichi Miyagawa, Yasuyuki Miyahara, Naoki Yoshihara, and participants of 9th international conference of Association for Public Economic Theory for helpful comments.
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due to the inequality of education caused by differences in their circumstances: An individual in a disadvantaged group may achieve lower level of education than one in an advantaged group even if the two individuals exert the same level of effort to acquire education. Roemer (1998), among others, insists that agents should not be held accountable for the differential backgrounds, and that social inequalities caused by the unaccountable differences in circumstances should be reduced. Governments may help disadvantaged individuals to reduce such inequalities due to factors for which agents should not be accountable.

Then, it would be important to study consequences of education policies derived from various criteria. In this paper, we consider this problem in a model of education planning, using tools of mechanism design.[1]

In this paper, we consider an economy where agents differ in two respects. First, agents may have different talents, which are relevant to their costs of acquiring education. Second, agents may belong to different social groups such as races or genders. We assume that the planner can observe each agent's group membership, but cannot observe his talent. Thus, the planner faces self-selection constraints with respect to agents' talents. For simplicity, we consider two levels of talents, high-talent and low-talent.

Second, we assume that there are two social groups, advantaged and disadvantaged groups. We assume that the relative proportion of high-talented agents in the disadvantaged group is smaller than the proportion in the advantaged group. This assumption expresses situations where people in a group socially discriminated against (due to gender, race, and so on) tend to pay larger (psychological) costs of obtaining education than people in other groups.[2]

We take into account two kinds of advantage from education: Each agent's own education level, and externality of education. Examples of benefit from each own education would contain enhancement of human capital and accumulation of knowledge. Examples of externality regarding education could include crime reduction, development of new technologies and knowledge, and increase in political awareness. Magnitudes of the externalities are different among countries. These magnitudes would be larger in developing countries than in developed countries. [3] We take account of magnitude of

[1]Our model is a modified version of Fleurbaey et al. (2002).

[2]See, for instance, Roemer (1998, pp. 5-12) for further discussions regarding this point.

[3]We refers to Hanushek (2002) for these discussions. Moreover, he discusses that re-

the externality.

The planner's policies are as follows: The planner distributes *income transfers* and *public spending on education* (called "schooling help" to reduce agents' costs) and determines agents' education levels. These policies are determined to maximize a social welfare function. we compare two allocations of education policies derived from two well-known social welfare functions, the *maximin* and the *utilitarian*. Note that the maximin social welfare function is often called the *Rawlsian*, since this is similar to the difference principle proposed by Rawls (1971).

Our main findings are as follows: When we use the utilitarian social welfare function, there is no difference among different groups. The optimal allocation of education levels and levels of public spending depends only on their talents. In contrast, when we consider the Rawlsian social welfare functions,

· for high-talented agents there is no difference among social groups, and

· for low-talented agents, agents in the disadvantaged group obtain more education.

Moreover, they receive more schooling help if the magnitude of the externality is sufficiently small. These results would mean that the Rawlsian education policy would lead to a kind of "reverse discrimination" among different social groups.

Affirmative action is a particular example of policies such that more preferential treatments may be given to agents in the more disadvantaged groups. In this paper, we actually show that education policies produce the above reverse discrimination without such particular policies. We also show that, in contrast, utilitarian education policies do not distinguish agents by their group memberships, although it is discussed that the utilitarinan planner is not concerned about this kind of equality.

1.1 Related literature

There are many researches on distribution of public expenditure on education. Arrow (1971) firstly considered the utilitarian approach to the problem of distributing public spending. Ulph (1977) and Hare and Ulph (1979) analyzed situations where both education and income redistribution policies are

ducing social inequalities and existence of externality of education are major justification of governments' interventions into education.

simultaneously performed. They considered the distribution problem under asymmetric information where agents' talents are unobservable. They used the framework of the optimal income taxation constructed by Mirrlees (1971). Fleurbaey et al. (2002) constructed the model we adopt. They investigated the distribution of public spending on education (schooling help) and income transfers when the planner has the egalitarian social welfare function of the CES type functions with various degrees of inequality aversion. Our model is an extension of theirs to the case where agents may belong to different social groups, and there exists an externality of education.

There are also studies taking into account externalities of education. Green and Sheshinski (1975) extended Arrow's (1971) model to the case where all agents benefit from the aggregate education level, as well as their own education levels. They showed that, if there exists the externality of education, The utilitarian social planner shifts public expenditures to the high talented individuals who can use the resources efficiently. Then, the planner increases the positive externality. De Fraja (2002) also considered the same externality of education as Green and Sheshinski. He constructed a mechanism for optimal education policies in the situation where the households differ in their income and in unobservable talents of their children. He adopted the utilitarian social welfare function. His model allows for the existence of a private education sector.

The structure of the paper is as follows. In section 2, we introduce the model. In section 3, as a benchmark, we analyze the case where agents' talents are observable. In section 4, we study the distribution problem under incomplete information. In section 5, we provide some concluding remarks.

2 The model

We consider an economy where the whole population is continuum which is divided into four types, according to talents $\{\theta_H, \theta_L\}$ and two social characteristics $j = 1, 2$. Each type ij denotes a group of agents with a talent θ_i and a social characteristic j. Each agents choose an effort level $x \geq 0$ to acquire education. A talent θ is a marginal disutility of effort. We assume $\theta_2 > \theta_1 > 0$, which means that agents with θ_H are more talented than ones with θ_L. Let $p_{ij} > 0$ be the probability of type ij $(i = H, L,\ j = 1, 2)$. We assume that

$$\frac{p_{H1}}{p_{L1}} < \frac{p_{H2}}{p_{L2}}.$$

This means that group 1 is more disadvantaged than group 2.

Public spending on education called "schooling help", s, is an in-kind transfer provided by the public sector. It is assumed that an education production function g gives the educational achievement of an individual as an increasing function of effort and help. Let $e = g(x, s)$ be the education level an agent of an agent with talent θ when exerting effort x and receiving help s.

We denote

$$E = \sum_{i=H,L} \sum_{j=1,2} p_{ij} e_{ij}$$

be the externality of education in the society when the education level of type ij is e_{ij}.

Each agent's gain from education is given by a real valued function $B(e, E)$. Note that each agent receives benefit from social education level as well as his own private education level. For simplicity, we assume that $B(e, E) = e + aE$, where a is a magnitude level of the externality.

The effort expenditure of talent θ is θx. The minimal amount of effort needed to achieve the education level while receiving s is denoted $x = C(e, s)$. By definition, $e \equiv g[C(e, s), s]$. For the needs of analysis, we assume the following,

Assumption. The mapping C is twice continuously differentiable, with partial derivatives satisfying
(1) $C_e > 0,\ C_s < 0,\ C_{es} < 0$;
(2) $C_e \to 0$ as $e \to 0$ for all s; $C_s \to 0$ as $s \to +\infty$, and $C_s \to -\infty$ as $s \to 0$ for all e;
(3) C is strictly convex;
(4) $C_{ee} > |C_{es}|$ and $C_{SS} > |C_{es}|$.

Assumption (1) implies that an additional education increases the cost, an additional help decreases the cost, and an additional help decreases the marginal cost of education. Assumption (2) is the Inada condition to ensure the interiority of education and help for simplicity of the analysis. Assumption (3) says that the returns to scale in the production of an individual level of education are strictly decreasing. Assumption (4) is to ensure the second order condition for the maximization problems below.

Each agent receives a monetary transfer $t \in \mathbb{R}$ from the public sector. We assume type ij's utility, u_{ij}, is given as follows:

$$u_{ij} = u(e_{ij}, s_{ij}, t_{ij}, E) = t_{ij} + e_{ij} + aE - \theta_i C(e_{ij}, s_{ij}).$$

We consider the two social welfare function is *utilitarian*;

$$\sum_{i=H,L} \sum_{j=1,2} p_{ij} u_{ij},$$

and the *Rawlsian*;

$$\min_{i=H,L} \min_{j=1,2} u_{ij}.$$

3 Distribution under Complete Information

As a benchmark, we consider the case where agents' talents are observable. We compare the optimal allocations with respect to the utilitarian and the Rawlsian social welfare functions. The problems are

$$\max_{(t,e,s)} \sum_{i=H,L} \sum_{j=1,2} p_{ij} u_{ij} \quad \text{and} \quad \min_{i=H,L} \min_{j=1,2} u_{ij}, \tag{1}$$

subject to the *budget balance* constraint (BB)

$$\sum_{i=H,L} \sum_{j=1,2} p_{ij}(t_{ij} + s_{ij}) = M, \tag{2}$$

where M is an amount of money which is exogenously given.

Let (e^{CR}, s^{CR}, t^{CR}) and (e^{CU}, s^{CU}, t^{CU}) be the solutions of the Rawlsian and the utilitarian social welfare functions, respectively, under complete information. We can easily obtain that the first order conditions for these social welfare functions are the same

$$1 + a = \theta_i C_e(e_{ij}^{Ck}, s_{ij}^{Ck}), \tag{3}$$

$$1 = \theta_i C_s(e_{ij}^{Ck}, s_{ij}^{Ck}), \tag{4}$$

$$u_{H1} = u_{H2} = u_{L1} = u_{L2} \text{ in the case of the Rawlsian,} \tag{5}$$

$$\sum_{i=H,L} \sum_{j=1,2} p_{ij}(t_{ij}^{Ck} + s_{ij}^{Ck}) = M, \tag{6}$$

where $k = U, R$, $i = H, L$ and $j = 1, 2$. Moreover, with the Rawlsian social welfare function, the utility levels of all types are the same. Since the same first order conditions (3) and (4) are derived from these social welfare functions, $(e^{CR}, s^{CR}) = (e^{CU}, s^{CU})$. Then, let $(e^C, s^C) \equiv (e^{CR}, s^{CR}) = (e^{CU}, s^{CU})$.

Let $(\theta, a) \mapsto (e^C(\theta, a), s^C(\theta, a))$ be the solution mappings of the above problem. We obtain the following result. [4]

Propostion 1. Suppose that Assumption (1)-(4) hold.

(i) $e^C(\theta, a)$ is decreasing in θ.

(ii) $s^C(\theta, a)$ is increasing in θ if the impact of the externality a is small enough that $a < -(C_{ee} + C_{es})/C_{es}$.

Proof. Differentiating (3) and (4) by θ and solving the system of the equations, we obtain

$$e^C_\theta(\theta, a) = \frac{-1}{d(C'')}[(1+a)C_{ss} + C_{se}]\frac{1}{\theta^2}, \quad (7)$$

$$s^C_\theta(\theta, a) = \frac{1}{d(C'')}[(1+a)C_{es} + C_{ee}]\frac{1}{\theta^2}, \quad (8)$$

where $d(C'') = C_{ee}C_{ss} - (C_{es})^2 > 0$ and $j = 1, 2$. On equation (6), by Assumption (4), $e^C_\theta(\theta, a) < 0$.

On equation (7), note that $C_{es} < 0$. Thus, $s^C_\theta(\theta, a) > 0$ if $(1+a)C_{es} + C_{ee} > 0$ at the solution. □

From proposition 1, $e^C_{Hj} > e^C_{Lj}$ $(j = 1, 2)$. Moreover, $s^C_{Lj} > s^C_{Hj}$ if $a < -(C_{ee} + C_{es})/C_{es}$ $(j = 1, 2)$. Note that, if the impact of the externality a is sufficiently large, high-talented agents receive higher levels of help. An intuition behind this fact is that, if the impact of the externality is sufficiently large, then the planner has incentive to reduce high-talented agents' cost and to let them acquire more education to increase the externality.

Next, note that the left-hand-side of equations (3) and (4) are common between the two social groups. Thus, we obtain the following result.

Proposition 2. Under complete information, the allocation of education and help is the same between the two groups 1 and 2. That is, $(e^C_{i1}, s^C_{i1}) = (e^C_{i2}, s^C_{i2})$ for $i = H, L$.

Therefore, when the planner can observe the talents of the agents, there are *no discrimination* with respect to education between the two social groups.

[4] The result is almost the same as Propositon 1 of Fleurbaey et al. (2002), exept that we allow for the externality of education.

4 Distribution under Incomplete Information

In this section, we analyze the case where the planner cannot observe agents' talents. Note that the planner can observe agents' group memberships.

We introduce *Incentive Compatibility* constraints (IC_{ij}) ($i = H, L$ and $j = 1, 2$);

$$
\begin{aligned}
&(IC_H) \quad u_{Hj} \geq t_{Lj} + e_{Lj} + aE - \theta_H C(e_{Lj}, s_{Lj}), \\
&(IC_L) \quad u_{Lj} \geq t_{Hj} + e_{Hj} + aE - \theta_L C(e_{Hj}, s_{Hj}) \;\; (j = 1, 2).
\end{aligned}
$$

Then, our problem is

$$\max_{(t,e,s)} \sum_{i=H,L} \sum_{j=1,2} p_{ij} u_{ij} \quad \text{and} \quad \min_{i=H,L} \min_{j=1,2} u_{ij},$$

subject to (BB), (IC_H) and (IC_L).

It is convenient to rewrite the three constraints in terms of (u, e, s). It is easy to obtain the following constraints.

(BB'): $\quad M = \sum_{i=H,L} \sum_{j=1,2} p_{ij} \left[s_{ij} + u_{ij} - e_{ij} - aE + \theta_i C(e_{ij}, s_{ij}) \right].$

(IC'_{Hj}): $\quad u_{Hj} \geq u_{Lj} + (\theta_L - \theta_H) C(e_{Lj}, s_{Lj})$ ($j = 1, 2$).

(IC'_{Lj}): $\quad u_{Lj} \geq u_{Hj} - (\theta_L - \theta_H) C(e_{Hj}, s_{Hj})$ ($j = 1, 2$).

4.1 The utilitarian solution

When using the utilitarian social welfare function, it is easily shown that the incentive compatibility constraints do not bind. Let (e^u, s^u) be the second best utilitarian solution regarding education. The first order conditions are as follows.

$$1 + a = \theta_i C_e(e^u_{ij}, s^u_{ij}), \tag{9}$$

$$1 = \theta_i C_s(e^u_{ij}, s^u_{ij}), \tag{10}$$

where $i = H, L$ and $j = 1, 2$.

We can see that, in this case, the first order conditions (equations (9) and (10)) are the same as those under complete information (see also Fleurbaey et al. (2002), section 3.3). Therefore, $(e^u, s^u) = (e^C, s^C)$. Then, since the first-best allocation of education and help, (e^C, s^C), does not distinguish agents by their group memberships, the same result holds in the allocation of the utilitarian education policies under incomplete information. We summarize the result as follows:

Proposition 3. Suppose that Assumption (1)-(4) hold. Under incomplete information, the utilitarian education policy does not discriminate the different social groups in the following sense: $e_{i1} = e_{i2}$ and $s_{i1} = s_{i2}$ for $i = H, L$.

Thus, when the planner adopts the utilitarian social welfare function, the agents with the same talent receive the same levels of education and help, irrelevantly of their group membership.

4.2 The Rawlsian solution

When using the Rawlsian social welfare function, by the same discussion as Fleurbaey et al. (2002) (section 3.3), we can show that the above three constraints can be reformulated as follows. First, incentive compatibility constraints for high-talented agents are binding;

$$(IC'_{Hj}): \quad u_{Hj} = u_{Lj} + (\theta_L - \theta_H)C(e_{Lj}, s_{Lj}) \text{ for } j = 1, 2.$$

Second, (IC_L) can be rewritten as follows;

$$(IC'_{Lj}): \quad C(e_{Hj}, s_{Hj}) \geq C(e_{Lj}, s_{Lj}) \text{ for } j = 1, 2.$$

It can be checked that (IC_{L_j}) does not bind.

Finally, from (BB), $\sum_{i=H,L} \sum_{j=1,2} p_{ij}(t_{ij} + s_{ij}) = M$, and the form of utility, $u_{ij} = t_{ij} + e_{ij} + aE - \theta_i C(e_{ij}, s_{ij})$, we obtain;

$$(BB'): \quad M = \sum_{i=H,L} \sum_{j=1,2} p_{ij} \left[s_{ij} + u_{ij} - e_{ij} - aE + \theta_i C(e_{ij}, s_{ij}) \right].$$

Hence, we have obtained the reformulated constraints, (IC'_{H_j}), (IC'_{L_j}) $(j = 1, 2)$ and (BB').

We ignore (IC'_{Lj}) $(j = 1, 2)$, as the usual methodology in the literature of mechanism design. One can check that these constraints do not bind. Then, by the Rawlsian social welfare function, (IC_{Hj}) for $j = 1, 2$, and the assumption that the social group memberships are observable for the planner, we can see that $u_{L1} = u_{L2} = u_L$ and the planner maximizes u_L subject to the constraints above. Substituting IC_{Hj} $(j = 1, 2)$ into BB', we obtain the following equation:

$$4u_L = \sum_{i=H,L} \sum_{j=1,2} p_{ij} \left[e_{ij} + aE - \theta_i C(e_{ij}, s_{ij}) - s_{ij} \right] - \sum_{j=1,2} p_{Hj}(\theta_L - \theta_H) C(e_{Lj}, s_{Lj}).$$

The second-best optimal solution can be obtained by maximizing the right-hand side of the above equation. Let (e^r, s^r) be the second-best allocation concerning education. The first order conditions are as follows: For $j = 1, 2$,

$$\frac{1+a}{\theta_H} = C_e(e^r_{Hj}, s^r_{Hj}), \tag{11}$$

$$\frac{-1}{\theta_H} = C_s(e^r_{Hj}, s^r_{Hj}), \tag{12}$$

$$\frac{1+a}{\theta_L + (\theta_L - \theta_H) p_{Hj}/p_{Lj}} = C_e(e^r_{Lj}, s^r_{Lj}), \tag{13}$$

$$\frac{-1}{\theta_L + (\theta_L - \theta_H) p_{Hj}/p_{Lj}} = C_s(e^r_{Lj}, s^r_{Lj}). \tag{14}$$

By equations (11) and (12), there is no distortion in the allocation of high-talented agents. That is, $e^r_{Hj} = e^C_{Hj}$, $s^r_{Hj} = s^C_{Hj}$ $(j = 1, 2)$.

On the other hand, according to equations (13) and (14), low-talented agents are treated as if they have even lower talent $\theta_L + (\theta_L - \theta_H) p_{Hj}/p_{Lj}$ in equations (3) and (4). Remember that $p_{H1}/p_{L1} < p_{H2}/p_{L2}$ by assumption. Since $e^C_\theta < 0$ by Proposition 1-(i), equations (13) and (14) imply

$$e^r_{L1} = e^C(\theta_L + (\theta_L - \theta_H)\frac{p_{H1}}{p_{L1}}, a) > e^C(\theta_L + (\theta_L - \theta_H)\frac{p_{H2}}{p_{L2}}, a) = e^r_{L2}.$$

Moreover, Proposition 1-(ii) ($s^C_\theta < 0$ if $(1 + a)C_{es} + C_{ee} > 0$) implies that,

$$s^r_{L1} = s^C(\theta_L + (\theta_L - \theta_H)\frac{p_{H1}}{p_{L1}}, a) > \text{ (resp. } <) s^C(\theta_L + (\theta_L - \theta_H)\frac{p_{H2}}{p_{L2}}, a) = s^r_{L2}$$

if $(1 + a)C_{es}(e^r_{Lj}, s^r_{Lj}) + C_{ee}(e^r_{Lj}, s^r_{Lj}) > 0$ (resp. < 0) for $j = 1, 2$.

In sum, we have obtained the following result:

Proposition 4. Suppose that Assumption (1)-(4) hold. Under incomplete information, the second-best policy regarding education derived from the Rawlsian social welfare function has the following properties.

(i) $e^r_{H1} = e^r_{H2}$, $s^r_{H1} = s^r_{H2}$.

(ii) $e^r_{L1} > e^r_{L2}$.

(iii) $s^r_{L1} >$ (resp. $<$) s^r_{L2} if $(1 + a)C_{es} + C_{ee} > 0$ (resp. < 0).

(i) implies that the high-talented agents in the two groups are not discriminated with respect to education. They obtain the same levels of education and help.

(ii) means that the low-talented agents in the disadvantaged group achieve higher education level than the ones in the advantaged group. This would be interpreted as a form of reverse discrimination, since the more disadvantaged group is given more preferential treatment than the advantaged group.

(iii) mentions that the low-talented agents in the disadvantaged agents receive higher level of help than the ones in the advantaged group whenever the degree of externality or $|C_{es}|$ is sufficiently small.

5 Concluding Remarks

In this paper, we have shown that the facts below hold when the social planner cannot observe the agents' talents. First, the Rawlsian education policy leads to a sort of "reverse discrimination" in the following sense: The low-talented agents in the advantaged group achieve the lower level of education than the ones in the disadvantaged group, and the former receives less education help than the latter if the magnitude of the externality is sufficiently small (Proposition 4). Note that the high-talented agents' the levels of education and help are the same between the two groups. Second, in contrast, the education policies derived from the utilitarian social welfare function do not distinguish agents by their group memberships (Proposition 3).

An intuition behind the results is as follows. Firstly, note that the Rawlsian social welfare function tends to equalize *utilities* among agents. Then, the planner would require the high-talented agents to achieve the high level of education, and transfer income from the high-talented ones to the low-talented ones. Under asymmetric information, by the incentive compatibility constraints, the planner must give the information rent to the high-talented ones in order to make them exerts the efficient level of effort (see the constraint IC_H). To reduce the information rent, the planner would lower the cost of the low-talented ones. To do so, the planner could let the low-talented ones exert lower effort, and give them more help, in the second-best allocation than in the first-best. In equation (13) and (14), the low-talented agents in the advantaged group is treated as if they have lower talent than the ones in the disadvantaged group. Therefore, the results follow.

There would be, at least, two possible ways to avoid the "reverse discrimination". The first is that the planner does not distinguish agents in the different groups. Then, the allocation becomes the same between the two groups. The second is to consider some non-welfaristic social welfare functions. It remains for future researched to consider extensions in this direction.

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