Growth or Welfare State? Optimal Composition of the Government Expenditure*

Yusuke KINAI†

Graduate School of Economics, Osaka University

September, 2010
<Very preliminary and Incomplete>

Abstract

In developing countries, the government often has to promote at once growth-enhancing and generational distribution policy. However, under such a situation, it is difficult for the government to carry out both policies and therefore, it is unavoidable to determine which policy they should carry out or the allocation of tax revenue through political compromise. Then, this paper incorporates both pay-as-you-go type pension and public investment into an overlapping generations model and shows how the contents of economic policy varies under the situation in which the policy determination is dependent on political issue.

Keywords: Commitment ; Growth-enhancing vs. Redistribution Scheme; Overlapping Generations Model; Structure-induced Equilibrium.

JEL Classification: E61, H54, H55.

*The author is thankful to Kazuo Mino and Masatsugu Tsuji for their variable suggestions. Comments from Akihiko Kaneko, Gregory Gilpin, Nobuo Akai, Keigo Kameda, Hiroki Tanaka and the seminar participants at PET10 held at Boğazişi University and Kansai Public Economics Seminar are also greatly acknowledged. The usual disclaimer applies.

†Corresponding to: Graduate School of Economics, Osaka University, 1-7, Toyonaka, Osaka, 560-0043, Japan
E-mail : ykinai@js8.so-net.ne.jp

1 Introduction

Traditionally, in analyzing economic policies such as fiscal or monetary policy, economists have assumed that the government is a monolithic organization that is intended mainly to maximize social welfare or the economic growth rate. In this regard, the following points should be emphasized: First, various entities (voters, bureaucrats, representatives, organizations, etc.) that are involved in policy determination foster conflicts. Therefore, it is impossible for the government to organize a policy based on only one situation or position. In other words, the government cannot avoid determining a policy that incorporates implications of numerous opinions. Second, even if a policy were derived that maximizes social welfare or the growth rate, carrying out such a policy with certainty would be difficult: it is hard to commit to such a policy. Some conflicts exist even within actual governments. As just one example, the Ministry of Finance and the Ministry of Health, Labor and Welfare in Japan have frequent conflicts. These two organizations seek different objectives: the former seeks to decrease the deficit or debt, whereas the latter is responsible for promoting a social security system, even if it is very costly. Consequently, a contraposition of duties pertains between the two organizations. As a result, we cannot regard actual governments as monolithic organizations, as many economists have assumed. Nevertheless, many studies assume that policy variable is one-dimensional. Given the existence of such conflicts, "political compromise" is inevitable; it is difficult to maintain the government commitment entirely. As countermeasures for such a situation, we find that it is necessary *ex post* to carry out some kind of coordination policy to attenuate inefficiencies that result from discretionary policy. Then, as a source of such conflicts, we can point that the policy determination is not necessarily one dimensional but often multi-dimensional. Therefore, it is necessary to expand the past analysis into multi-dimensional policy-determination. This paper specifically considers the situation in which the government has two kinds of policy options; pension and public investment.

In this paper, we specifically consider the situation in which there exit two kinds of committee in the government and one is related to public investment and the other is pension system. Under such a situation, how does the content of the policy determine ? This paper is an attempt to answer such a question. The present paper analyzes how the contents of the policy varies through time, assuming that policy is determined based on the result of voting.

Then, let us consider the case of Japan. Fig.1 depicts the flow of the proportion of pension and public investment to gross domestic production[1)]. From this figure, we find that the cost of pension

[1)] Data Source:
Social Security : "the costs of social security" given by SNA; *The National Institute of Population and Social Security Research.*

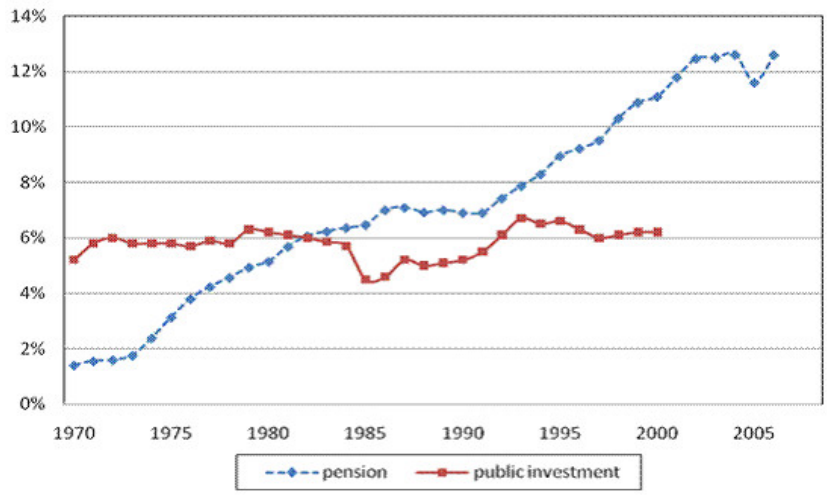

Figure 1　The flow of the proportion of pension and public investment to gross domestic production in JAPAN

has been increasing, while the cost of public investment remains unchanged. One of the purposes of this paper is to give an explanation to the fact that the cost of pension has been increasing rather than public investment as in Japan. What is the reason why the cost of the one policy is higher than the other? This paper shows that pension is politically more favourable than public investment as a result of capital accumulation.

■Relationship with the Past Studies　Let us explain the position of the present paper in the literature. Since the seminal paper such as Barro (1990), Futagami, Morita and Shibata (1993) or Alesina and Rodrik (1994), there are many studies those analyze the relationship between public investment and economic growth[2)]. But there are not so many studies those analyze the relationship in an overlapping generations (hereafter, OLG) model. As starting studies, we can point out Pestieau (1974), or Yoshida (1986). Since their studies, there are some studies as follows: Among others, for instance, Yakita (1994) and Burgess (2006) investigate the relationship between the return of public investment and the discount factor. Glomm and Ravikumar (1997) introduce the public investment into an OLG model in a framework of endogenous growth model. Recently, Kaas (2003) incorporates Majority Voting regarding the policy determination and show the existence of cyclical equilibria as for the tax rate. But policy determination of his model is one-dimensional. Moreover, Hung (2005) focuses on the monetary aspect by introducing seigniorage. More recently, Yakita (2008) expands two-dimensional policy determination by incorporating not only public investment but also maintenance activity into an OLG model, but he does not consider the political issue regarding the policy determination.

Public Investment(Government gross fixed capital formation)：*SNA*：*System of National Accounts Statistics*

2) As a survey, see Irmen and Kühnel (2008).

On the other hand, regarding studies those analyze pension policy in an OLG model, there exit numerous studies, but our survey is limited to the analysis in a context of political economy. For instance, we can point out Casamatta, Cremer and Pestieau (2000), Wigger (1999), Razin, Sadka and Swagel (2002), Aíbo, Mahieu and Patxot (2004), Boldrin and Rustichini (2000) and so forth. What is common to these studies is that policy determination is one dimensional, although these are close to this paper in the respect that policy determination is conducted though voting.

As shown the above, there exist lots of studies those analyze the effect of pension or public investment in an OLG model. However, to the best of our knowledge, there are few studies those incorporate both pension and public investment into an OLG model[3)]. As an exception, we can point out Maebayashi (2010) who introduces pension into the model of Yakita (2008), but his result is limited to the corner solution. In detail, Maebayashi shows that the government allocates total tax revenue to public investment if they aim to maximize the growth rate. As other study, although Creedy, Li and Moslehi (2008) expands the past studies into two-dimensional policies; pension and public goods provision, they limit the analysis to the balanced growth path.

Contrary to those studies, the feature of this study is summarized as follows: First, we expand policy determination to the two-dimensional policy determination by introducing pension into the model of Kaas (2003), who incorporates public investment into an OLG model. Second, we regard the public capital as stock variable, while Kaas does as flow variable. That is because we focus on the transitional path as well as balanced-growth-path. Third, this paper considers political issue regarding policy determination, which differs from Yakita (2008). Additionally, note that the model of this paper does not apply ordinal voting theory because policy determination in our model is two-dimensional, and we resolve such a difficulty by employing the concept of structure-induced equilibrium developed by Shepsle (1979).

The rest of this paper is structured as follows: In the section 2, we set up the model, and analyze the optimal taxation in the case that there is no political issue in the section 3. In the section 4, we analyze the situation in which policy determination is dependent on the voting behavior. Final remarks is described in the section 5.

[3)] However, there exist some studies focusing on the combination of pension and education. For instance, see Naito (2009), Boldrin and Montes (2005), Lambrecht, Michel and Vidal (2005), and Kaganovich and Zilcha (1999). These studies focus on the combination of BIG (Backward Intergenerational Goods) and FIG (Forward Intergenerational Goods) using the term in Rangel (2003).

2 The Model

We employee the Diamond (1965)-type two-period overlapping generations model without bequest motive in a closed-economy. There is a population growth, that is, $N_{t+1} = (1+\mu)N_t$, in which μ can be both positive and negative. Time is discrete and goes to infinity. We incorporate social security policy into the model of Kaas (2003). Therefore, the government has two kinds of policies; pension and public investment. We introduce the heterogeneity of households, that is, households varies depending on the labor productivity. We also consider the political issue regarding the policy determination as in Kaas (2003). The difference between Kaas (2003) and the present paper is that we expand his analysis into two-dimensional policy determination. In what follows, we consider the case in which there is no political issue (voting behavior) in the section 3 as a benchmark, while in the section 4, we consider the case in which there is political issue.

2.1 Behaviours

■Households In what follows, we call households those who are born in t period as generation t. The generation t solves the following problem:

$$\max_{s_t^i} \; U(c_t^{yi}, c_{t+1}^{oi}) \tag{1}$$

Here, we impose some assumptions on the utility function, following Kaas (2003) or Kaas and von Thadden (2003). First, $U(\cdot)$ is twice differentiable with respect to each variable, homothetic, strictly increasing, and quasi-concave. Second, with respect to each variable, the utility function is homogeneous degree of $1-\gamma$, in which $\gamma \in [0,1]$ denotes the degrees of relative risk aversion ($\gamma \equiv -\dfrac{cu''}{u'}$). Third, consumption in young period and old period is substitute.

There exist two kinds of tax: pension tax (τ_t) and public investment tax (θ_t), those are imposed on both labor-income and saving. Households those form at period t divide after-tax labor-income ($\tilde{w}_t$) into consumption (c_t^y) and saving (s_t), and consume (c_{t+1}^o) as after-tax savings and pension (d_t) when they are old. Consequently, the young-period and old-period budget constraints are shown respectively as

$$c_t^{yi} + s_t^i = \tilde{w}_t l_t^i, \quad c_{t+1}^{oi} = \tilde{R}_{t+1} s_t^i + d_{t+1}, \tag{2}$$

where $\tilde{w}_t \equiv (1-\tau_t-\theta_t)w_t$ and $\tilde{R}_{t+1} \equiv (1-\tau_{t+1}-\theta_{t+1})R_{t+1}$. Each household determines their own saving by solving the following problem.

$$\max_{s_t^i} \; U(-s_t^i + \tilde{w}_t l_t^i, \tilde{R}_{t+1} s_t^i + d_{t+1})$$

By solving the above equation, the saving rate ($s_t(\cdot)^i$) is only dependent on $\tilde{R}_{t+1}$ and the saving function is denoted as the multiplicity separable as follows:[4].

$$s_t^i = s^i(\tilde{R}_{t+1})\tilde{w}_t l_t^i \tag{3}$$

By substituting this into the utility function, the indirect function is denoted as

$$V(\cdot) = U(1 - s_t^i(\tilde{R}_t), s_t^i(\tilde{R}_t)\tilde{R}_{t+1})\tilde{w}_t^{\gamma-1}. \tag{4}$$

Note that the following equation holds, because the consumption in young and old period is assumed to be substitute:

$$\frac{\partial s(\cdot)}{\partial \tilde{R}_{t+1}} \geq 0. \tag{5}$$

We then assume that the economy is *dynamically efficient*, that is, the following equation holds:

Assumption 1

$$R_t > 1 + \mu. \tag{6}$$

This equation means that the return of saving is larger than that of pension and the condition, which eliminates the trivial situation in which all households support pension system.

■The Government Next, we turn to the government's behavior. they have two kinds of scheme: public investment scheme (growth-enhancing scheme) and the PAYG-type pension system (intergenerational redistribution scheme). They determine which policy they should carry out, or the allocation of tax revenue based on the voting. The budget constraint of each scheme is balanced in each period. Tax revenue of public investment and pension respectively as E_t and M_t. Note that the policy variables are θ_t (contribution to public investment) and τ_t (contribution to pension). Both schemes are covered with labor- and capital-income taxes. In what follows, we normalize the total labor input as 1, that is, $\sum_i l_t^i \equiv L_t = 1$.

Growth-enhancing scheme:

The government covers the cost of public investment with capital- and labor-income taxes as in Barro (1990). We assume public investment has no congestion effect. Therefore, the budget constraint is written as

$$E_t = \theta_t Y_t = \sum_i \theta_t w_t l_t^i + \theta_t R_t s_{t-1} = \theta_t(w_t L_t + R_t s_{t-1}) = \theta_t(w_t + R_t s_{t-1}) \tag{7}$$

where G_t and E_t denote public capital stock and public investment, respectively. The evolution of public investment is written as

$$G_{t+1} - (1 - \delta_G)G_t = E_t. \tag{8}$$

[4] Regarding the reason why the saving rate is dependent only on R_{t+1}, not w_t, see De La Croix and Michel (2002, pp.53–54). In case that the utility function is homothetic, the saving function can be denoted as multiplicity separable.

Arranging the above equation into per capita, we have

$$(1+\mu)g_{t+1} = e_t + (1-\delta_G)g_t, \tag{9}$$

where δ_g denotes the depression rate of public capital.

Intergenerational redistribution scheme:

Next, we move to the explanation of pension as a role of intergenerational redistribution. In aggregate, noting that the pension system is pay-as-you-go type, the budget constraint of this scheme is written as

$$N_{t-1}d_t = \tau_t Y_t = \tau_t R_t s_{t-1} + \sum_i \tau_t w_t l_t^i = \tau_t(w_t L_t + R_t s_{t-1}) = \tau_t(w_t + R_t s_{t-1}) = M_t \tag{10}$$

Dividing the both sides of the above equation with N_t, we obtain

$$\underbrace{d_t}_{\text{pension received by households}} = (1+\mu)\underbrace{\tau_t(w_t + R_t s_{t-1})}_{\text{contribution}} \equiv M_t. \tag{11}$$

Merging two schemes, the budget constrain of the government is written as

$$E_t + M_t = (1+\mu)(\theta_t + \tau_t)(w_t L_t + R_t s_{t-1}) = (1+\mu)(\theta_t + \tau_t)(w_t + R_t s_{t-1}).$$

■Firms We then describe the firms' behavior. We assume that factor markets are perfectly competitive and that firms maximise their profits. We here specify the production function as follows:

$$Y_t = F(K_t, A_t L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

Here, we define A_t as

$$A_t \equiv a\left(\frac{K_t^{\beta} G_t^{1-\beta}}{L_t}\right), \quad \beta \in [0,1] \tag{12}$$

where G_t denotes public capital. In words, as in Yakita (2008) and Kaas (2003), we assume that public investment has a positive externality in a sense that it takes up the marginal labor productivity. Note that the model of Kaas (2003) corresponds to the case of $\beta = 0$ in eq.(12). Using this definition, the production function is rewritten as

$$Y_t = K_t^{\alpha+\beta(1-\alpha)} G_t^{(1-\beta)(1-\alpha)} = K_t^{\omega} G_t^{1-\omega}, \tag{13}$$

where $\omega \equiv \alpha + \beta(1-\alpha)$. Firms are assumed to solve the following problem:

$$\max\ \Pi = F(K_t, A_t L_t) - R_t K_t - w_t L_t$$

where K_t, L_t, Y_t denote respectively capital, labor , output in aggregate. Let us define $k_t \equiv \frac{K_t}{A_t L_t}$, and $F(\frac{K_t}{A_t L_t}, 1) = f(k_t)$. Then, by solving the above profit maximization problem, we can obtain as follows:

$$R_t = A_t f'(k_t) = \alpha \left(\frac{K_t}{G_t}\right)^{\omega-1} \equiv R(\theta_t) \tag{14a}$$

$$w_t = A_t \{\underbrace{f(k_t) - k_t f'(k_t)}_{w(k_t)}\} = (1-\alpha)\left(\frac{K_t}{G_t}\right)^{-\omega}\left(\frac{G_t}{L_t}\right) \equiv w(\theta_t), \tag{14b}$$

Note that these two factor prices are function of public investment tax, because G_t is financed with $theta_t$. Taking eqs. (14b), (7), (12) and (8) into consideration, we have

$$w(k_t;\theta) = \frac{1}{a\theta_t}\left(\frac{G_t}{K_t}\right)^{\beta}. \tag{15}$$

Here, taking τ_t and θ_t as given, $F(\cdot)$ can be rewritten as

$$F(K_t, A_t L_t) = A_t L_t f(k_t) = \frac{f(k_t)}{k_t} K_t. \tag{16}$$

Therefore, we can rewrite the production function, $F(\cdot)$ as AK-type production function taking the policy variables as given. Moreover, net labor income can be written in a form of linear function of capital;

$$w_t(1-\tau_t-\theta_t)L_t = \tilde{w}_t L_t \frac{w(k_t)}{k_t}(1-\tau_t-\theta_t)K_t = \Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)K_t, \tag{17}$$

where $\Psi(\theta_t;\tau_t) \equiv \dfrac{1}{a\tau_t k(\theta_t)}\left(\frac{G_t}{K_t}\right)^{\beta}$. Note that τ_t is given. Moreover, we assume $\Psi(\cdot)$ satisfies the following:

Assumption 2 There exits only one solution, θ^* which maximizes $\Psi(\theta_t;\tau_t)$,

which means the condition that ensures the existence of the solution.

2.2 Market Equilibrium

Finally, let us describe the equilibrium condition of each market.

(1) Capital Market

In aggregate,

$$\sum_i s_t^i(\cdot)\tilde{w}_t l_t^i = K_{t+1} \tag{18}$$

From eq.(17), considering the saving rate; $s_t(\tilde{R}_t)$, we have

$$K_{t+1} = \sum_i s^i(R(\tau_{t+1}))\Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)K_t \tag{19}$$

Transforming the above equation into per capita, we obtain

$$k_{t+1} = \overline{s(R(\theta_{t+1}))}\Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)k_t. \tag{20}$$

Here, $\overline{s(\tilde{R}(\theta_{t+1}))}$ denote the average of saving rate.

To summarise, the capital market clearing condition is written as

$$k_{t+1} = \frac{1}{1+\mu} \underbrace{s(\tilde{R}(\theta_{t+1})}_{\text{saving rate}} \tilde{w}(k_t) l_t^i. \tag{21}$$

(2) Goods Market

In aggregate, we can state this condition as follows:

$$c_t^y N_t + c_t^o N_{t-1} + (K_{t+1} - K_t) + (G_{t+1} - (1-\delta_G)G_t) + M_t = Y_t$$

Dividing both sides of the above equation with N_t yields:

$$c_t^y + \frac{c_t^o}{1+\mu} + m_t + (1+\mu)(k_{t+1} + g_{t+1}) = R_t k_t + w_t l_t + (1-\delta_G) g_t \tag{22}$$

(3) Labor Market

Denoting labor demand as L_t, the condition is:

$$N_t = L_t. \tag{23}$$

Here, let us describe the definition of the competitive equilibrium.

Defnition 1 *Taking K_0 and G_0 as given, we define $\{c_t^y, c_t^o, s_t, k_t, l_t, R_t, w_t, g_t, \theta_t, \tau_t\}$ as competitive equilibrium such that*

1. *For all t, Taking $\{R_t, w_t, \theta_t, \tau_t\}$ as given, the condition of utility maximization for generation t holds.*
2. *For all t, Taking $\{\tau_t, \theta_t\}$ as given, the condition of profit maximization holds.*
3. *Taking $\{c_t^y, c_t^o, s_t, k_t, l_t, R_t, w_t\}$ as given, $\{G_t, E_t, M_t, \theta_t, \tau_t\}$ meets the budget constraint of the government.*
4. *All markets clear.*

2.3 The Timing of Decision Making

The timing of decision making in t period, is summarized as follows, which is also depicted in Fig. 2.

Stage 1. Households are born in t period.

Stage 2. Households vote over the two policy variables; contribution to pension; τ_t and the one to public investment; θ_t in young period.

Stage 3. Firms produce.

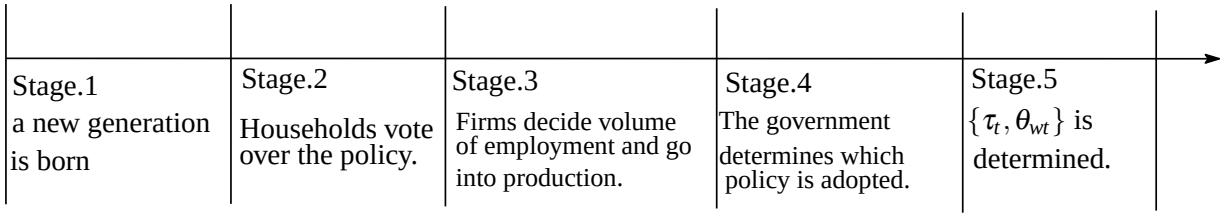

Figure 2 Sequence of Decision Making in t-th period.

Stage 4. Based on the result of voting, the government determines the allocation of tax revenue (or which scheme the government admits).

Stage 5. A new generation $t+1$ are born in the next period.

3 Policy Determination without Political Process

In this section, we assume the case in which there is no political issue as a benchmark. In other words, we treat the policy determination as a solution of maximization problem. We consider the case in which the objective function of each scheme is growth rate and the social welfare.

3.1 Growth-Rate-Maximization Tax Rate

In what follows, we limit our analysis to the Balanced-Growth-Path. Then, we here seek the growth rate at balanced growth path. The growth rate of each variable is as follows:

Substituting the saving function, eq.(3) into the capital market-clearing condition, eq.(**??**), and using eqs.(14b), (8),(7), we have

$$\frac{K_{t+1}}{K_t} = \frac{s(\tilde{R}(\tau_{t+1}))\Psi(\tau,\theta)(1-\tau-\theta)}{K_t} \tag{24a}$$

$$\frac{G_{t+1}}{G_t} = (1-\delta_G) + \frac{\tau_t w_t L_t}{G_t} \tag{24b}$$

Define $\gamma = \frac{\frac{K_{t+1}}{N_{t+1}}}{\frac{K_t}{N_t}} = \frac{\frac{G_{t+1}}{N_{t+1}}}{\frac{G_t}{N_t}} = \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}}$. Then, the intersection of the above two equation; eqs.(24a) and (24b) is BGP.

Letting x_t be $\dfrac{K_t}{G_t}$, we then investigate the sign of $|\frac{dx_{t+1}}{dx_t}|$ and its slope. We then have

$$\frac{dx_{t+1}}{dx_t} > 0,$$

and

$$\lim_{x_t \to \infty} \frac{dx_{t+1}}{dx_t} = \infty.$$

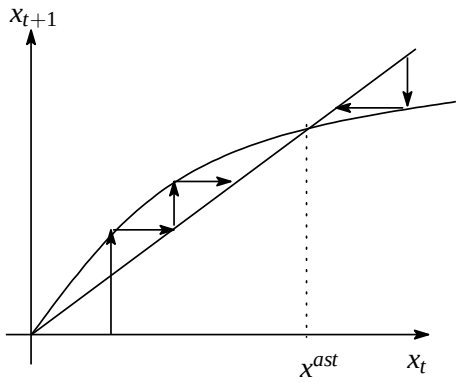

Figure 3 Determination of Balanced Growth Path

Therefore, we then can state this result in a form of lemma.

Lemma 1 *There exists at least one BGP.*

Then, differentiating BGP with respect to τ and *theta*, we can derive the growth-rate-maximizing tax. Let us define τ and θ as τ^* and θ^* such that

$$\frac{d\gamma}{d\tau} = 0 \tag{25}$$

$$\frac{d\gamma}{d\theta} = 0 \tag{26}$$

3.2 Welfare-Maximization Tax Rate

Define the objective function as follows:

$$V^i(R_t) = s^i(\tilde{R}_{t+1})\tilde{w}_t^{1-\gamma} \tag{27}$$

$$\frac{\partial V(\cdot)^i}{\partial \theta} = 0 \tag{28}$$

$$\frac{\partial V(\cdot)^i}{\partial \tau} = 0 \tag{29}$$

By solving the above equations, we can derive the welfare-maximizing tax τ^{**}, and θ^{**}.

Comparison of Tax Rate Then, let us investigate the relationship between the growth-rate maximizing and welfare-maximizing tax rate. The following proposition answers such a question:

Proposition 1 *The relationship between growth-rate-maximizing and social-welfare maximizing tax rate is given by*

$$\tau^* \neq \tau^{**}, \quad \theta^* \neq \theta^{**}$$

Intuitively, the reason is explained as follows: If the government aims to maximize the growth-rate, they try to allocate the tax revenue as much as possible to public investment which has a role of promoting economic growth. Therefore, the growth-rate maximizing tax is higher than welfare-maximizing one.

This result differs from that of Barro (1990). In Barro (1990), he claimed that the tax rate that maximizes the growth rate equals to that maximzies the social welfare. But this result shows that his claim is not robust.

4 Policy Determination with Political Process

4.1 Equilibrium Concept and Some Assumptions

Given the discussion in the previous section, we advance the analysis by endogenizing policy determination. In this section, we introduce the political issue (i.e., voting behavior). Here, note that there are two policy variables in our model. It is known that, generally, *no* Condorcet winner does not exist in voting over multiple issues such as a combination of policy of two kinds, without imposing additional conditions on voter's preference[5)]. To avoid such a problem, following Conde-Ruiz and Galasso (2005), we adopt the concept of a structure-induced equilibrium developed by Shepsle (1979)[6)].

Let us consider the following situation. There exist two kinds of committee: One is the committee which determines the contribution to pension and the other is the contribution to public investment. The policy determination itself is achieved independently, and the one policy determination is based on the other policy determination. Namely, this situation can be regarded as the state in which there exit following two reaction functions:

$$\begin{cases} \tau = \tau(\theta) & : \text{Taking } \theta \text{ as given.} \\ \theta = \theta(\tau) & : \text{Taking } \tau \text{ as given.} \end{cases}$$

and we regard the intersection of the above two response functions as an (politico-economic) equilibrium.

We then consider the voting behavior related to determination of contribution to pension and public investment. We here assume that

A 1. Voting is held in each period, which means issue-by-issue voting under direct democracy.

A 2. Voting takes place *simultaneously* on contributions to pension and public investment.

[5)] Regarding this issue, see Persson and Tabellini (2000), for instance.

[6)] Regarding the studies those who employee the structure-induced equilibrium, see table 1. This paper differs from those studies in the sense that they focus on the combination of income redistribution schemes, whereas this paper focuses on the combination of social security policy and other kind of economic policy.

Table1　Past studies those use the concept of structure-induced equilibrium

Conde-Ruiz and Galasso (2005)	Class of Social Security
	PAYG-type pension vs. Redistribution Policy
Poutvaara (2006)	Class of Social Security
	Pension vs. Public Education
Conde-Ruiz and Profeta (2007)	Type of Pension
	Bismarckian vs. Beveridean
Konishi (2008)	Financial Resource of Social Security
	Consumption Tax vs. Labor-income Tax
Bethencourt and Galasso (2008)	Class of Social Security
	Pension vs. Health Investment
Kinai (2008)	Class of Social Security
	PAYG-type Pension vs. Unemployment Insurance
This Paper	Class of Economic Policy
	Public Investment vs. PAYG-type Pension

A 3. Voters consist of young and old people who are alive in the same period.

A 4. Policy determination is based on majority voting.

A 5. Voting is repeated among successive generations of voters.

Before entering into the analysis, we have to show the following lemma:

Lemma 2 *Defining $V(\cdot)$ as the indirect utility function, the following equations hold:*

$$\frac{\partial^2 V(\cdot)}{\partial \theta^2} < 0, \qquad \frac{\partial^2 \gamma}{\partial \tau^2} < 0$$

Proof *We can obtain the indirect utility function as follows:*

$$\begin{aligned} V(\cdot) &= U(1 - s_t(R_t), s_t(R_t)\tilde{R}_{t+1}) I_t^{\gamma - 1} \\ &= s^i(R_{t+1})\tilde{w}_t \end{aligned} \tag{30}$$

From this equation, by differentiating twice, we find,

$$\frac{\partial^2 V(\cdot)}{\partial \tau^2} < 0.$$

∎

This lemma shows that the indirect utility function is concave and therefore, we can employee the median voter theorem regarding policy determination.

■Determination of contribution of public investment

- We employee the Median Voter Theorem[7].
- As in the previous analysis, the indirect utility function is derived as follows:

$$\begin{aligned} V(\cdot) &= U(1-s_t(R_t), s_t(R_t)\tilde{R}_{t+1})I_t^{\gamma-1} \\ &= s^i(R_{t+1})\tilde{w}_t \end{aligned} \tag{31}$$

Applying the Median Voter Theorem to the policy determination of contribution to public investment,

$$\frac{\partial V(\cdot)}{\partial \tau} = 0 \tag{32}$$

From this equation, this relationship can be written as

$$\tau^{med} = \tau^{med}(\theta)$$

■Determination of Contribution to Pension System We also employee the Median Voter Theorem. The indirect utility function is written as:

$$\begin{aligned} V(\cdot) &= U(1-s_t(R_t), s_t(R_t)\tilde{R}_{t+1})I_t^{\gamma-1} \\ &= s^i(R_{t+1})\tilde{w}_t \end{aligned} \tag{33}$$

Differentiating with respect to θ, we obtain,

$$\frac{\partial V(\cdot)}{\partial \theta} = 0$$

The preference of median voter is:

$$\frac{\partial V(R_t)^{med}}{\partial \theta} = 0 \tag{34}$$

Solving this equation yields:

$$\theta^{med} = \theta(\tau) \tag{35}$$

Here, note that we can show that the two reaction functn is downward-sloping by totally diffrentiating.

4.2 The Case with Commitment

First, let us consider the case with commitment. That is, we assume the tax rate is constant over the time; $\theta = \theta_t = \theta_{t+1}$, $\tau = \tau_t = \tau_{t+1}$. Here note that the shape of the two response functions; $\tau = \tau(\theta)$ and $\theta = \theta(\tau)$ are down-sloping. Then, there are three plausible cases as depicted in Fig 4 $\sim$ 6: To summarize the above discussion,

[7] In this respect, our approach is similar to that of Alesina and Rodrik (1994).

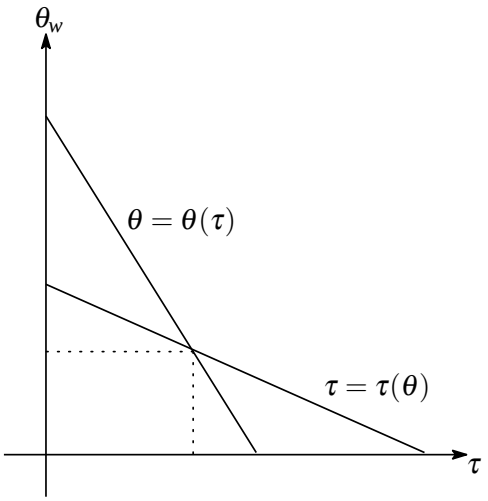

Figure 4 Case 1.

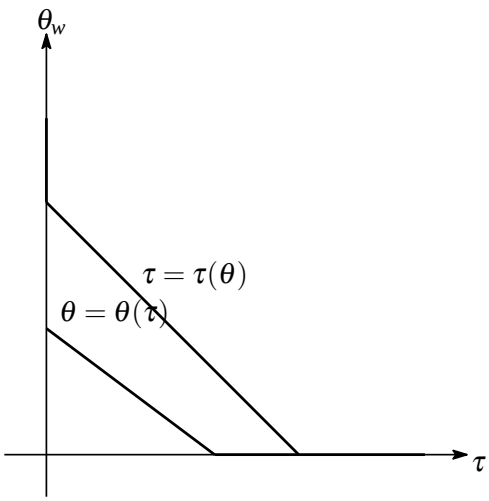

Figure 5 Case 2.

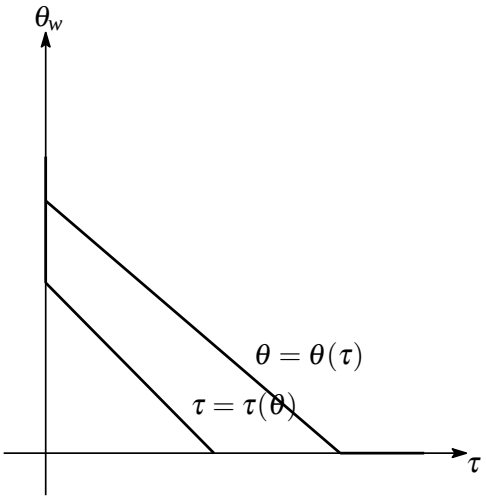

Figure 6 Case 3.

Proposition 2 *Depending on the parameters, there are three plausible cases:*

1. *Both policies (pension and public investment) survive.*
2. *Only pension survives.*
3. *Only public investment survives.*

4.3 The Case without Commitment

Then, we go to the analysis under no commitment. Before entering into analysis, let us define the game structure of our model. We then formally define the voting game. The public history of

the game at t period, $h_t = \{(\tau_0, \theta_{w0}), (\tau_1, \theta_1), ..., (\tau_{t-1}, \theta_{t-1})\} \in H_t$ is the sequence of social security system (pension and public investment). H_t is the set of all possible history at time t. An action profile for those who support pension is, $\{\tau_t, b_t\} \in [0,1] \times [0,1]$. Analogously, an action for unemployed individual at time t is $\{\tau_t, b_t\} \in [0,1] \times [0,1]$.

Then, a strategy for those who support pension at t period is a mapping from the history of the game into the action space, that is, $\sigma^e : h_t \to \{\tau_t, \theta_t\}$. Analogously, a strategy for those who support public investment is at t period is $\sigma^u : h_t \to \{\tau_t, \theta_{wt}\}$. The strategy profile played by both individuals at t period is denoted by $\sigma_t \equiv \sigma_t^e \cup \sigma_t^u$.

At t periods, the objective function for young each player is

$$V_t^i(\sigma_0^i, \sigma_1^i, ...\sigma_t^i, \sigma_{t+1}^i ...) = V_t^i(\tau_t, \theta_{wt}, \tau_{t+1}, \theta_{w,t+1}).$$

and regarding agents those who support public investment,

$$V_t(\sigma_0, \sigma_1, ...\sigma_t, \sigma_{t+1} ...) = V_t^i(\tau_t, \theta_{wt}).$$

These solutions describe the relationship between the policy at t period and the one at $t+1$ period.

Then, lets us describe the definition of equilibrium.

Defnition 2 *(The Definition of Markovian Structure-Induced Equilibrium)*

1. *σ meets the property of Markov perfect equilibrium.*
2. *For all t, at t period, the equilibrium outcome associated to σ_t is a structure-induced equilibrium of the static game with commitment.*

As contrasted with the analysis in the previous subsection, we assume that the government has no commitment technology in this subsection. Then, let us define the history of the game H_t as

$$H_t^0 \equiv \{h_t \in H_t | \theta_t = \theta_w *, \ \ t \in \{0, 1, ...\}\},$$

and

$$H_t^\sigma \equiv \{h_t \in H_t | \theta_k = 0, \ k = 0, 1, ...t_0, \ and \ \theta_{wt} = 0, \ t \geq t_0.\}.$$

Moreover, the strategy profiles of people those who support pension and public investment are respectively denoted as σ_t^e and σ_t^u. We then investigate whether each player has an incentive to deviate from the solution under full commitment, as discussed in the previous subsection. Under this setting, we first verify that unemployed people have no an incentive to deviate from the strategy. We assume that unemployed people adopt the following strategy: $\theta_{t0}^{deviate} > \theta_w^*$ and $\tau_{t0}^* < \tau_t^{deviate}$. However, employed people do not obtain an additional payoff by deviation because they punish the other by reducing the payment of contributions to the pension system, τ, which exerts negative effects on the welfare of both

agents. Therefore, it is apparent that unemployed people do not have an incentive to deviate from the commitment solution.

Regarding people those who support public investment, presuming that those who support public investment deviate from equilibrium, i.e. they avoid paying contributions to pensions, then the workers will punish the other by not paying contributions to public investment. They would pay contributions to pensions to avoid being punished. Therefore, it is apparent that they have no incentive to deviate. To summarize, neither those who support pension nor public investment people have an incentive to deviate.

From the discussion, we have:

Proposition 3

Policies discussed in the previous subsection (with commitment case) coincide with those without commitment. In other words, the strategies with commitment are time-consistent.

5 Concluding Remarks

In this paper, we consider the case in which the government has two kinds of policy; pension system and public investment. In our setting, policy determination is based on majority voting. The government has social security policy mechanisms of two kinds: pension and public investment. Under this setting, we show how the contents of economic policy varies.

References

Aíbo, G., G. Mahieu, and C. Patxot (2004) "On the optimality of PAYG pension systems in an endogenous fertility setting," *Journal of Pension Economics and Finance*, Vol. 3, No. 01, pp. 35-62.

Alesina, Alberto and Dani Rodrik (1994) "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, Vol. 109, No. 2, pp. 465-490.

Barro, Robert J (1990) "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, Vol. 98, No. 5, pp. S103-26.

Bethencourt, Carlos and Vincenzo Galasso (2008) "Political Complements in the welfare state: Health Care and Social Security," *Journal of Public Economics*, Vol. 92, No. 3-4, pp. 609-632.

Boldrin, Michele and Ana Montes (2005) "The Intergenerational State: Public Education and Pensions," *Review of Economic Studies*, Vol. 72, No. 3, pp. 651-664.

Boldrin, Michele and Aldo Rustichini (2000) "Political Equilibria with Social Security," *Review of Economic Dynamics*, Vol. 3, pp. 41-78.

Burgess, David (2006) "Public Investment Criteria in Overlapping Generations Models of Open Economies," *International Tax and Public Finance*, Vol. 13, No. 1, pp. 59-78.

Casamatta, Georges, Helmuth Cremer, and Pierre Pestieau (2000) "The Political Economy of Social Security," *Scandinavian Journal of Economics*, Vol. 102, No. 3, pp. 503-522.

Conde-Ruiz, J. Ignacio and Vincenzo Galasso (2005) "Positive Arithmetic of the Welfare State," *Journal of Public Economics*, Vol. 89, No. 5-6, pp. 933-955.

Conde-Ruiz, J. Ignacio and Paola Profeta (2007) "The Redistributive Design of Social Security Systems," *Economic Journal*, Vol. 117, No. 520, pp. 686-712.

Creedy, John, Shuyun May Li, and Solmaz Moslehi (2008) "The Composition of Government Expenditure in an Overlapping Generations Model," Department of Economics - Working Papers Series 1043, The University of Melbourne.

De La Croix, David and Philippe Michel (2002) *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*: Cambridge University Press.

Diamond, Peter A. (1965) "National Debt in a Neoclassical Growth Model," *American Economic Review*, Vol. 55, No. 5, pp. 1126-1150.

Futagami, Koichi, Yuichi Morita, and Akihisa Shibata (1993) "Dynamic Analysis of an Endogenous Growth Model with Public Capital," *Scandinavian Journal of Economics*, Vol. 95, No. 4, pp. 607-25.

Glomm, Gerhard and B. Ravikumar (1997) "Productive government expenditures and long-run growth," *Journal of Economic Dynamics and Control*, Vol. 21, No. 1, pp. 183-204.

Hung, Fu-Sheng (2005) "Optimal Composition of Government Public Capital Financing," *Journal of Macroeconomics*, Vol. 27, No. 4, pp. 704-723.

Irmen, Andreas and Johanna Kühnel (2008) "Productive Government Expenditure and Economic Growth," Working Papers 0464, University of Heidelberg, Department of Economics.

Kaas, Leo (2003) "Productive Government Spending, Growth, and Sequential Voting," *European Journal of Political Economy*, Vol. 19, No. 2, pp. 227-246.

Kaas, Leo and Leopold von Thadden (2003) "Unemployment, Factor Substitution and Capital Formation," *German Economic Review*, Vol. 4, pp. 475-495.

Kaganovich, Michael and Itzhak Zilcha (1999) "Education, social security, and growth," *Journal of Public Economics*, Vol. 71, No. 2, pp. 289-309.

Konishi, Hideki (2008) "Financing Social Security by Consumption Tax: a Political Economy Perspective." mimeo, Tokyo Institute of Technology.

Lambrecht, Stephane, Philippe Michel, and Jean-Pierre Vidal (2005) "Public pensions and growth," *European Economic Review*, Vol. 49, No. 5, pp. 1261-1281.

Maebayashi, Noritaka (2010) "Public Capital, Public Pension, and Growth," Discussion Papers in Economics and Business 10-03, Osaka University, Graduate School of Economics and Osaka School of International Public Policy (OSIPP).

Naito, Katsuyuki (2009) "From the Cradle to the Grave? A Politico-economic Approach on Social Security and Public Pension." Kyoto University, mimeo.

Persson, Torsten and Guido Tabellini (2000) *Political Economics: Explaining Economic Policy*: MIT Press.

Pestieau, Pierre (1974) "Optimal taxation and discount rate for public investment in a growth setting," *Journal of Public Economics*, Vol. 3, No. 3, pp. 217-235.

Poutvaara, Panu (2006) "On the Political Economy of Social Security and Public Education," *Journal of Population Economics*, Vol. 19, No. 2, pp. 345-365.

Rangel, Antonio (2003) "Forward and Backward Intergenerational Goods: Why is Social Security Good for the Environment?" *American Economic Review*, Vol. 93, No. 3, pp. 813-834.

Razin, Assaf, Efraim Sadka, and Phillip Swagel (2002) "The Ageing Population and the Size of the Welfare State," *Journal of Political Economy*, Vol. 110, No. 4, pp. 900-918.

Shepsle, Kenneth (1979) "Institutional Arrangements and Equilibrium in Multidimensional Voting Models," *American Journal of Political Science*, Vol. 23, No. 1, pp. 27-59.

Wigger, Berthold U. (1999) “Pay-as-you-go financed public pensions in a model of endogenous growth and fertility,” *Journal of Population Economics*, Vol. 12, No. 4, pp. 625-640.

Yakita, Akira (1994) “Public investment criterion with distorted capital markets in an overlapping generations economy,” *Journal of Macroeconomics*, Vol. 16, No. 4, pp. 715-728.

——— (2008) “Ageing and public capital accumulation,” *International Tax and Public Finance*, Vol. 15, No. 5, pp. 582-598.

Yoshida, Masatoshi (1986) “Public Investment Criterion in an Overlapping Generations Economy,” *Economica*, Vol. 53, No. 210, pp. 247-63.