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Analysis on the Demand of Education: Modeling One's Prospects through Education.

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Abstract

While the role of education as an investment, a signal, or a consumption is well known, this paper introduces another important aspect of education that it provides people with prospects for their future. Through education, people are able to obtain information about their own abilities or aptitudes, which allows them to choose fields where they can excel. This paper models this property of education in two manners: One is to treat it using a Bayesian probability and the other is to treat it *a priori*. It is shown that an educational demand depends not only on what one can expect from acquiring a particular skill, but also on the degree of development in the region. The theory is then applied to examine the effect of economic development on the demand of education, using both Japanese cross-section data and international panel data. The results are consistent with the prediction from the model, particularly for the early stages of education.

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1 Introduction

It is widely believed that education, or human capital accumulation, is essential for the economic development or economic growth in a region. In fact, much research has been carried out, following this viewpoint, in examining the effect of education on economic development both from a theoretical and empirical approach. For example, Lucas (1988) has theoretically described the role of human capital for economic growth in its well-known endogenous growth model. Mankiw, Romer, and Weil (1992) have examined cross-country growth differences using neoclassical growth model where they have included a human capital. Krueger and Lindahl (2001) is an excellent survey on empirical studies concerning the effects of education on economic development. Haveman and Wolfe (1984) examine some educational externalities that might have positive effects on economic development. However, surprisingly few studies so far have been made in examining the effect of economic development on education, even though the innovation of new technology or knowledge which stems from the economic development should influence the decision of individuals on the demand of education. One of the reasons for the lack of studies in this direction is due, we believe, to the fact that there has been no theory for education to address this problem.

This paper introduces a fairly new view of education, for the purpose of addressing the problem. While the role of education as an investment[1], a signal[2], or a consumption[3] are well known, this paper insists that education has another important aspect that, through education, people are informed of their expected outcomes of future education. In the traditional theory for education, it is assumed that the educational outcomes are known in advance. However, in reality, they are unknown since people are not known of their innate abilities or aptitudes in advance, and also due to the fact that educational outcomes may change by accident. Instead, people obtain information about their own abilities or aptitudes through education, which makes it possible for them to predict their outcomes of their future education, and therefore to work in fields in which they can perform best. In fact,

[1]Becker (1964) and Schultz (1963) are famous studies that discuss the role of education as an investment.

[2]Spence (1973) and Arrow (1973) point out the signaling aspect of education. Stiglitz (1976) discusses the screening aspect of education which resembles the role of education as a signal.

[3]See for example, discussion in Gullason (1989)

some elementary school students learn quite a wide range of subjects such as mathematics, baseball, and piano not necessarily in the hope of being a mathematician, a professional baseball player, or a pianist but rather in the hope of finding a field they excel in, as they can not know in advance.

We present two types of models that theoretically manage this property of education. One model treats this property of education using a Bayesian probability. This model describes the process wherein people acquire skills, evaluate their innate abilities, and decide their career paths. We allow several types of skills to coexist and the level of each skill to vary between individuals. Given the price of each skill, each worker determines the supply of each skill. The demand side of the skills is also examined to derive the general equilibrium of the model. We see that the Bayesian probability obtained thorough education is important in the decision of the career path. Our finding is that the variety of skills available is relevant to the educational preferences of people. The other model, on the other hand, assumes the prospective property of education a priori. Though the role of education is assumed exogenously in the model, its advantage is that it makes it easy for us to extract the effect of economic development on the demand of education. These two models make us understand how the uncertainty or potential an individual faces for the future is important for the decision on educational demand. The main implication our theory presents, is that an educational demand depends not only on what one can expect from acquiring a particular skill but also on the possibility or uncertainty one faces, one of which is the degree of development in a region. This result is examined empirically using Japanese cross-section data and international panel data. It is shown that the theory fits very well to the statistical facts especially for the early stages of education.

This paper is organized as follows. A model is presented in Section 2, where the prospects individual obtains through education is described using a Bayesian probability, exploiting a 3-period, 3-skill, 3-job model. The effect of economic development on the demand of education is analyzed in Section 3 using a 2-period, n-skill, 1-job model which is a somewhat simple model discussed in Section 2. The findings drawn from the theoretical form is empirically examined in Section 4 using both Japanese prefectural data and international panel data. The conclusion is presented in Section 5.

2 3-Period, 3-Skill, 3-Job Model

Supply Side of the Model

The analysis begins by constructing a three-period overlapping generations model. In this economy, L (a constant) individuals are born in every period, and each individual lives for three periods. In each period, individuals allocate their time to work or skill acquisition (education). Individuals can acquire a skill only by spending a period of time to learn it. Thus, to work for a particular job, an individual must first acquire the skill needed for the job. Assume that individuals are risk-neutral, and that the intertemporal discount rate is 0. Hence, individuals are only interested in their expected lifetime income.[4]

Let there be three types of skill A, B, and C. It is supposed that individuals are faced with the uncertainty of educational outcomes, in which the productivity of each skill is expressed by a random variable α ($0 \leq \alpha \leq M_A$), β ($0 \leq \beta \leq M_B$), and γ ($0 \leq \gamma \leq M_C$), respectively. The price of each skill per unit productivity in the labor market is denoted by p_A, p_B, and p_C, respectively. At the outset, individuals know the joint density function of α, β, and γ, i.e, $f(\alpha, \beta, \gamma)$, but not their own values. They will know their own level of skill with certainty only after they have spent time acquiring each skill.

An important point to notice is that, as long as α, β, and γ are correlated, knowing the value of α, for example, will allow an individual to infer his productivity in skill B and C. In fact, after learning skill A, the individual will revise his prior distribution over β to a conditional density function $f(\beta|\alpha)$. Thus, he obtains information about his innate traits by learning skill A. Note that skill acquisition provides information that decreases the uncertainty of each individual and hence makes the individual productive. In this way, Bayesian probability expresses the role of education that it provides individuals with prospects for their future in this model.

To make the analysis simple, let us limit our analysis to the case where all individuals start their career by learning skill A.[5] In the second period, then, each individual has three options. One is to begin a job career in

[4] As also assumed later, since individuals are not allowed to save (so that capital does not exist in the economy), there is no need to think about the interest rate.

[5] We can verify whether this behavior of individuals is justified as the equilibrium state afterward. Or, we may suppose a situation where people receive compulsory education.

the occupation where skill A is used for inputs. (We call it occupation A. The same applies for occupations B and C.) In this case, the individual receives $p_A\alpha$ (his productivity times the price of skill A) as a wage for the successive two periods; thus, earnings are $2p_A\alpha$ for a lifetime. Otherwise, the individual can spend time to acquire either skill B or C in the second period. Individuals who spend time to acquire skill B (C) in the second period can work at either occupation A or B (C) in the third period. Lifetime earnings are $p_A\alpha$ and $p_B\beta$ ($p_C\gamma$) in each case. Since they only have three periods, individuals never waste time learning a skill in the third period.

Figure 1 shows a branching diagram for five possible skill acquisition paths. Individuals choose which skill path to take, depending upon their realized value of skills, population distribution of the level of three skills (α, β, γ), and the price vector (p_A, p_B, p_C).

Note that the model can be interpreted in a different way. In reality, firms often determine the task that a worker will perform. The firm often trains workers to acquire a basic skill, evaluates their ability, and assigns them to a department where they seem to perform best. Thus, this sorting process is also consistent with the allocation choices of a profit-maximizing firm.

We can describe a sorting process by solving the decision-making problem backward. There are two nodes that require decision-making in the third period (after learning skill B or C in the second period). After the values of α and β (γ) are realized, the decision-making problem that the individual faces in the third period can be described as

$$\max\{p_A\alpha, p_B\beta\} \ \ (\ \max\{p_A\alpha, p_C\gamma\} \ ). \tag{1}$$

If $p_A\alpha$ exceeds $p_B\beta$ ($p_C\gamma$), the individual decides to work for occupation A; otherwise, one works for occupation B (C).

On the basis of the decision-making in the third period, each individual in the second period compares $2p_A\alpha$ to the expected value of learning skill B and skill C, given the value of α. Using the notation V_B and V_C to denote the value of learning skill B and skill C respectively, the decision-making problem in the second period can be described as

$$\max\{2p_A\alpha, V_B, V_C\}, \tag{2}$$

where

$$V_B = E(\max\{p_A\alpha, p_B\beta\}|\alpha), \text{ and} \tag{3}$$

$$V_C = E(\max\{p_A\alpha, p_C\gamma\}|\alpha). \tag{4}$$

The critical value of α can be found where the decision to work for occupation A is indifferent to the decision to learn skill B by equalizing $2p_A\alpha$ and V_B to get

$$2p_A\alpha = E(\max\{p_A\alpha, p_B\beta\}|\alpha) \tag{5}$$

or

$$2p_A\alpha = \int_0^{\frac{p_A}{p_B}\alpha} p_A\alpha f(\beta|\alpha)d\beta + \int_{\frac{p_A}{p_B}\alpha}^{M_B} p_B\beta f(\beta|\alpha)d\beta. \tag{6}$$

Rearranging equation (6) using $2p_A\alpha = 2p_A\alpha\Big(\int_0^{\frac{p_A}{p_B}\alpha} f(\beta|\alpha)d\beta + \int_{\frac{p_A}{p_B}\alpha}^{M_B} f(\beta|\alpha)d\beta\Big)$, we obtain

$$\alpha = \int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} (\hat{p}_B\beta - \alpha)f(\beta|\alpha)d\beta, \tag{7}$$

where we used the relative price $\hat{p}_B \equiv p_B/p_A$.

The left side of equation (7) expresses the cost of learning skill B: the foregone wage that could otherwise have been earned in occupation A. The right side represents the expected benefit of learning skill B. We can calculate equation (7) further by using the integration-by-parts formula to get

$$\alpha = \frac{1}{2}\Big(\hat{p}_B M_B - \int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} \hat{p}_B F(\beta|\alpha)d\beta\Big), \tag{8}$$

where we used $F(\beta|\alpha)$ which stands for the distribution function and the fact $F(M_B|\alpha) = 1$. Similarly, we can obtain

$$\alpha = \frac{1}{2}\Big(\hat{p}_C M_C - \int_{\frac{1}{\hat{p}_C}\alpha}^{M_C} \hat{p}_C F(\gamma|\alpha)d\gamma\Big) \tag{9}$$

by equalizing $2p_A\alpha$ and V_C where we again used the relative price $\hat{p}_C \equiv p_C/p_A$. Let us define the function on the right side of each equation (8) and (9) as $\hat{V}_B(\alpha)$ and $\hat{V}_C(\alpha)$, respectively.

While the shape of $\hat{V}_B(\alpha)$ depends on how skill A and B are correlated, we can ensure the existence of a solution in equation (8), which we summarize in the following proposition.

Proposition 2.1. If $F(\beta|\alpha)$ is continuous, there exists at least one solution in equation (8) in the interval $[0, \hat{p}_B M_B]$.

Proof. Since

$$\hat{V}_B(\hat{p}_B M_B) = \frac{1}{2}\hat{p}_B M_B, \quad \text{and}$$

$$\begin{aligned}\hat{V}_B(0) &= \frac{1}{2}\Big(\hat{p}_B M_B - \int_0^{M_B} \hat{p}_B F(\beta|0) d\beta\Big) \\ &\geq \frac{1}{2}\Big(\hat{p}_B M_B - \int_0^{M_B} \hat{p}_B d\beta\Big) = 0,\end{aligned}$$

the continuity of $\hat{V}_B(\alpha)$ guarantees at least one solution of equation (8). □

The same is true of the existence of a solution in equation (9).

The 45° line and two types of $\hat{V}_B(\alpha)$ are drawn in Figure 2 ($\hat{V}_{B_1}(\alpha)$ and $\hat{V}_{B_2}(\alpha)$). Since the 45° line and $\hat{V}_B(\alpha)$ represent the cost and the benefit of learning skill B, respectively, individuals will decide to learn skill B when $\hat{V}_B(\alpha)$ is above the 45° line, whereas, otherwise, they will decide to work for occupation A. Note that the form of $\hat{V}_B(\alpha)$ is an important factor to characterize the skill. In Figure 2, skill B_1 may be considered to be relatively simple, which is preferred by people with a low value of skill A. On the other hand, people who have acquired a high value of skill A tend to prefer skill B_2, which may be considered to be rather complicated. Thus, we can make sure that to learn skill A not only makes individuals possible to use the skill but also provides them with information about their aptitudes.

Using following definitions of notations,

$$\begin{aligned}\Delta_A &\equiv \{\alpha|\alpha \geq \hat{V}_B(\alpha), \alpha \geq \hat{V}_C(\alpha), \alpha \in [0, M_A]\} \\ \Delta_B &\equiv \{\alpha|\hat{V}_B(\alpha) \geq \alpha, \hat{V}_B(\alpha) \geq \hat{V}_C(\alpha), \alpha \in [0, M_A]\} \\ \Delta_C &\equiv \{\alpha|\hat{V}_C(\alpha) \geq \alpha, \hat{V}_C(\alpha) \geq \hat{V}_B(\alpha), \alpha \in [0, M_A]\},\end{aligned}$$

the decision of individuals is summarized in the second period and in the third period as the following lemmas.

Lemma 2.1. The solution of the decision-making problem individuals face in the second period, given the realized value of α, is as follows

(i). if $\alpha \in \Delta_A$, individuals decide to work for occupation A.

(ii). if $\alpha \in \Delta_B$, individuals decide to learn skill B.

(iii). if $\alpha \in \Delta_C$, individuals decide to learn skill C.

Lemma 2.2. The sorting process in the third period, after learning skill B or C, is described as follows.

(i). Individuals who learn skill B in the second period will decide to work for occupation B if $\hat{p}_B\beta \geq \alpha$, while decide to work for occupation A if $\alpha \geq \hat{p}_B\beta$.

(ii). Individuals who learn skill C in the second period will decide to work for occupation C if $\hat{p}_C\gamma \geq \alpha$, while decide to work for occupation A if $\alpha \geq \hat{p}_C\gamma$.

The notations L_A, L_B, and L_C are used to denote the population working in occupations A, B, and C, respectively. T_B and T_C refer to the population that is training to acquire skills B and C, respectively. Using Lemma 2.1, 2.2, and the marginal density function $f(\alpha)$, they can be described as follows.

$$\begin{aligned} L_A &= 2L\int_{\Delta_A} f(\alpha)d\alpha + L\int_{\Delta_B} f(\alpha)\int_0^{\frac{1}{\hat{p}_B}\alpha} f(\beta|\alpha)d\beta d\alpha \\ &+ L\int_{\Delta_C} f(\alpha)\int_0^{\frac{1}{\hat{p}_C}\alpha} f(\gamma|\alpha)d\gamma d\alpha \end{aligned} \tag{10}$$

$$L_B = L\int_{\Delta_B} f(\alpha)\int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} f(\beta|\alpha)d\beta d\alpha \tag{11}$$

$$L_C = L\int_{\Delta_C} f(\alpha)\int_{\frac{1}{\hat{p}_C}\alpha}^{M_C} f(\gamma|\alpha)d\gamma d\alpha \tag{12}$$

$$T_B = L\int_{\Delta_B} f(\alpha)d\alpha \tag{13}$$

$$T_C = L\int_{\Delta_C} f(\alpha)d\alpha. \tag{14}$$

Thus, the aggregate supply of each skill can be determined as

$$\begin{aligned} S_A &= 2L\int_{\Delta_A} \alpha f(\alpha)d\alpha + L\int_{\Delta_B} \alpha f(\alpha)\int_0^{\frac{1}{\hat{p}_B}\alpha} f(\beta|\alpha)d\beta d\alpha \\ &+ L\int_{\Delta_C} \alpha f(\alpha)\int_0^{\frac{1}{\hat{p}_C}\alpha} f(\gamma|\alpha)d\gamma d\alpha, \end{aligned} \tag{15}$$

$$S_B = L\int_{\Delta_B} f(\alpha)\int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} \beta f(\beta|\alpha)d\beta d\alpha, \quad \text{and} \tag{16}$$

$$S_C = L\int_{\Delta_C} f(\alpha)\int_{\frac{1}{\hat{p}_C}\alpha}^{M_C} \gamma f(\gamma|\alpha)d\gamma d\alpha. \tag{17}$$

Demand Side of the Model

The aggregate demand of skills is now determined by solving firms' cost-minimization problem. It is assumed for simplicity that, in each occupation, skills are transformed into middle-sector goods by one-to-one technology. The production in each occupation is denoted as m_A, m_B, and m_C. Each occupation is assumed to be completely competitive so that the price of goods produced in the occupation is the same as the price of a skill used for their inputs. In this economy, it is also assumed that there exists only one final goods whose inputs are the three middle-sector goods. Final goods are consumed immediately by individuals whose utility functions are linear to them and who are not allowed to save their earnings or products. The final goods are produced by firms that have an identical production function

$$Y = m_A^{\mu} m_B^{\nu} m_C^{1-\mu-\nu}, \tag{18}$$

The optimal behavior of each firm is to minimize their cost for any value of Y. We can determine the demand of firm i for each goods in period t, which we denote $D^i_{A,t}$, $D^i_{B,t}$, and $D^i_{C,t}$ by solving

$$\min_{m_{A,t}, m_{B,t}, m_{C,t}} p_{A,t} m_{A,t}(i) + p_{B,t} m_{B,t}(i) + p_{C,t} m_{C,t}(i)$$

$$s.t \;\; Y_t(i) = m_{A,t}(i)^{\mu} m_{B,t}(i)^{\nu} m_{C,t}(i)^{1-\mu-\nu}. \tag{19}$$

This results in the demand for each good to be simply given by

$$D^i_{A,t} = \lambda^i \frac{\mu}{p_A} Y_t(i) \tag{20}$$

$$D^i_{B,t} = \lambda^i \frac{\nu}{p_B} Y_t(i) \tag{21}$$

$$D^i_{C,t} = \lambda^i \frac{1-\mu-\nu}{p_C} Y_t(i), \tag{22}$$

where each firm assigns the same proportion of its sale (shadow price λ^i times output $Y(i)$) to each middle-sector goods.

The General Equilibrium

Finally, the general equilibrium of the model can be derived based on a supply and demand analysis. Using equations (15) $\sim$ (17) and equations (20) $\sim$ (22), the general equilibrium of the economy is determined in the following proposition.

Proposition 2.2.
$(\hat{p}_B^*, \hat{p}_C^*)$ is the relative price vector of the general equilibrium in the economy if and only if it satisfies the simultaneous equations

$$\hat{p}_B^* = \frac{\nu}{\mu}\frac{S_A(\hat{p}_B^*, \hat{p}_C^*)}{S_B(\hat{p}_B^*, \hat{p}_C^*)} \equiv \frac{\nu}{\mu}\frac{S_A^*}{S_B^*} \tag{23}$$

and

$$\hat{p}_C^* = \frac{1-\mu-\nu}{\mu}\frac{S_A(\hat{p}_B^*, \hat{p}_C^*)}{S_C(\hat{p}_B^*, \hat{p}_C^*)} \equiv \frac{1-\mu-\nu}{\mu}\frac{S_A^*}{S_C^*}. \tag{24}$$

Proof. (necessity condition) Since, in the equilibrium, the aggregate demand of each skill must be equal to the aggregate skill used for inputs in each occupation in each period, we get

$$D_A = \sum_i D_A^i = \sum_i \lambda^i \frac{\mu}{p_A} Y_t(i) = S_A \tag{25}$$

$$D_B = \sum_i D_B^i = \sum_i \lambda^i \frac{\nu}{p_B} Y_t(i) = S_B \tag{26}$$

$$D_C = \sum_i D_C^i = \sum_i \lambda^i \frac{1-\mu-\nu}{p_C} Y_t(i) = S_C. \tag{27}$$

Now, we can obtain equation (23) using (25) and (26), while equation (24) can be derived by using (25) and (27).
(sufficient condition) Since $\sum_i \lambda^i Y(i) = p_A D_A + p_B D_B + p_C D_C$, it can easily be shown that $D_A(\hat{p}_B^*, \hat{p}_C^*) = S_A^*$, $D_B(\hat{p}_B^*, \hat{p}_C^*) = S_B^*$, and $D_C(\hat{p}_B^*, \hat{p}_C^*) = S_C^*$ hold under the equilibrium price using equations (23) and (24). □

Note that there is no unemployment in this economy because individuals who want to work can always earn the marginal value products (i.e., the price of a skill times the amount of skill he offers). Thus, it is clear that the labor market-clearing condition is also satisfied in the equilibrium.

Numerical Examples

We now verify some characters of our economic model using numerical examples. It is shown that the variety of skills is relevant to the preferences among skills, essentially due to the property that education provides one with prospects for the future.

Let us make the model simpler in the way that each β and γ takes only two values, which means that individuals have only two results of learning skill B and C: Individuals either succeed in acquiring skill B (C) to get the productivity M_B (M_C) with probability $1 - u(\alpha)$ $(1 - v(\alpha))$ or fail to acquire skill B (C) and end up with productivity 0 with probability $u(\alpha)$ $(v(\alpha))$.

There are two objectives for setting functions as these. One is to be able to analyze the general equilibrium numerically, and the other is to clarify that skill B and C are rather specialized (or risky) skills compared to skill A, which we suppose to be a general-purpose skill. The joint distribution of skill productivities is, then, specified as follows.

$$f(\alpha,\beta,\gamma) = \begin{cases} u(\alpha)v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,0) \\ (1-u(\alpha))\,v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,0) \\ u(\alpha)\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,M_C) \\ (1-u(\alpha))\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,M_C) \\ 0 & \text{elsewhere} \end{cases}, \tag{28}$$

where we suppose $u(\alpha)$ to be linear, $v(\alpha)$ to be concave, and $\hat{f}(\alpha)$ to be the modified normal distribution. All the functions and the values of parameters used for the numerical analysis are collected in Appendix.

It is easy to confirm

$$f(\alpha) = \sum_{\gamma}\sum_{\beta} f(\alpha,\beta,\gamma) = \hat{f}(\alpha)$$

and

$$f(\beta|\alpha) = \begin{cases} u(\alpha) & (\beta = 0) \\ 1-u(\alpha) & (\beta = M_B) \end{cases} \tag{29}$$

$$f(\gamma|\alpha) = \begin{cases} v(\alpha) & (\gamma = 0) \\ 1-v(\alpha) & (\gamma = M_C) \end{cases} \tag{30}$$

Also notice that the concave property of function $v(\alpha)$ characterizes skill C to be more complicated skill, which requires a certain degree of basic ability to acquire. On the contrary, function $u(\alpha)$ characterizes skill B to be simpler, which is easy to acquire, even for individuals with less α[6](See Figure 3).

[6]It also assumes two other minor properties. (1) $u(\alpha)$ and $v(\alpha)$ are decreasing functions of α: Skill A and B (C) are positively correlated so that an individual with a high ability of general-purpose skills is more likely to succeed in acquiring skill B (C). (2) $u(0) =$

Figures 4 and 5 along with Table 1 collect the results that describe the state of this economy. Figure 4 shows $\hat{V}_B(\alpha), \hat{V}_C(\alpha)$, and the 45° line for the equilibrium skill price, where the information about the process of how individuals are sorted into distinct skill-learning paths is completely described. The proportion of people sorted into each skill acquisition path is described below Figure 1.

The equilibrium skill price of the model shown in Table 1 is the solution of the simultaneous equations (24) and (25). In addition, a proportion of L_A, L_B, L_C, T_B, and T_C within the labor force population (i.e. 2L) can be calculated by using equations (10) $\sim$ (14). The market-observed average wage and the latent average wage for a worker in each occupation are also collected in Table 1. The market-observed average wage refers to the wage that can be seen in the equilibrium market (i.e., $E_A, \hat{p}_B^* M_B$ and $\hat{p}_C^* M_C$, respectively), while the latent average wage refers to the expected wage of the individual if he were to work for each occupation after learning a corresponding skill. One point to notice is the inter-job wage differences in this economy. The average wages in occupation A and B (about 4) are observed to be far below the wage that workers in occupation C earn (about 50). This inequality essentially stems from the differences in ability among individuals and the sorting mechanism in the economy. Workers in occupation C earn much more than workers in occupation A and B not because they happened to choose to work at that particular job but because they were allowed to learn skill C and succeeded in acquiring the skill because they were talented.

It is also interesting to see how the distribution of basic skill ability will be reflected in the distribution of lifetime earnings in this economy. The distribution of lifetime earnings can be obtained by calculating it for each skill acquisition path. Figure 5.2, thus, describes how lifetime earnings I are distributed in this economy. We can notice that the inequality is increasing in the lifetime earnings compared to basic skill ability (Figure 5.1 and 5.2). Several studies point out that the variance in wages tends to be large compared to that in innate ability, which is consistent with our results.[7]

Finally, we can make sure that the individual in the first period does not

$v(0) = 1$, $u(b) = v(c) = 0$: An individual whose value of skill A is 0 always fails to acquire skill B (C), while an individual who has a value of skill A larger than b (c) will surely succeed in acquiring skill B (C).

[7]Roy(1951) is a classical study that shows insight into how some jobs would be the superior and attract superior laborers, whereas other jobs would be the inferior ones expanding inter-job wage differences when the abilities of skills are positively correlated.

have an incentive to deviate from learning skill A, and learn either skill B or C instead in our setting by calculating the expected lifetime income for each case (they are referred to as $E(I_A)$,$E(I_B^{DEV})$, and $E(I_C^{DEV})$).

It is clear that

$$E(I_A) = \frac{1}{L}\left(S_A^* + \hat{p}_B^* S_B^* + \hat{p}_C^* S_C^*\right). \tag{31}$$

By using a similar procedure to the one analyzed in previously, $E(I_B^{DEV})$ and $E(I_C^{DEV})$ can be verified and expressed as follows.

$$E(I_B^{DEV}) = \max\{W_{BA}, W_{BC}\} \tag{32}$$
$$E(I_C^{DEV}) = \max\{W_{CA}, W_{CB}\} \tag{33}$$

where

$$W_{BA} = \int_0^{M_\mathsf{A}} \alpha u(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_\mathsf{A}} \hat{p}_B^* M_B \left(1 - u(\alpha)\right)\hat{f}(\alpha)d\alpha, \tag{34}$$

$$W_{BC} = \int_0^{M_\mathsf{A}} \hat{p}_C^* M_C \left(1 - v(\alpha)\right) u(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_\mathsf{A}} \hat{p}_B^* M_B \left(1 - u(\alpha)\right)\hat{f}(\alpha)d\alpha, \tag{35}$$

$$W_{CA} = \int_0^{M_\mathsf{A}} \alpha v(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_\mathsf{A}} \hat{p}_C^* M_C \left(1 - v(\alpha)\right)\hat{f}(\alpha)d\alpha, \tag{36}$$

$$W_{CB} = \int_0^{M_\mathsf{A}} \hat{p}_B^* M_B \left(1 - u(\alpha)\right) v(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_\mathsf{A}} \hat{p}_C^* M_C \left(1 - v(\alpha)\right)\hat{f}(\alpha)d\alpha. \tag{37}$$

The results are collected in Table 2, where $E(I_A)$ has the largest value. Thus, we can justify the behavior that all individuals start from learning skill A is an equilibrium in this case.

We now turn to demonstrate that the skill availability (i.e., the number of skills that could be acquired in a country) is an important factor for the preference individuals hold. It can be shown that the existence of skill C may change people's preferences in their first period between skill A and B by comparing our 3-skill model to an economy in which only two skills (skill A and B) exist. The economic model is analyzed with two skills just as it was in the economy with three skills.[8] The results are in Table 2, which shows that people in 2-skill economy should have an incentive to abandon learning skill A and start their first period by learning skill B (shown as $E(I_A^{EQ})^{poor}$ and $E(I_B^{DEV})^{poor}$).

The preference relation between skill A and B in their first period has changed because skill A provides more information about one's innate traits compared to skill B in 3-skill economy due to the correlation between skill A and C. Generally, the more specialized skills there are, the more the value of learning general-purpose skill increases, as far as the skill has a correlation with specialized skills providing information about which skill to learn next. Obviously, this result would depend largely on the value of the parameters that were set. However, the point to notice is that the value of education depends not only on the outcome of learning a particular skill but also on the variety of skills in the region and the correlation among them. If there exist only two skills, people may prefer to learn more specific skill, while they may prefer to learn more general skill if there were more than three skills. We conclude this notion explaining the effect of economic development on the demand of education as in the following proposition.

Proposition 2.3. It can be that skill A is preferred to skill B in 3-skill region, whereas skill B is preferred to skill A in 2-skill region.

It, on the other hand, also tells us the possibility of differences in cross-country economic development. Some countries may develop more rapidly because people in these countries are more likely to acquire skills that are effective for economic development, while the opposite applies for other countries. If there is any reason to believe that the basic skill (or general-purpose skill) acquisition that is broadly correlated to other levels of education is the most influential on economic development[9], we believe that this notion casts

[8] To be exact, some devices in equation (18) are necessary for the analysis to be feasible. We may suppose that there is another production technology where skill A and B are the only inputs and where firms can choose whichever technology they want to use. Or, we may suppose that firms can obtain m_C with fixed cost $\bar{p}_C$ outside the country, both case maintaining the relation that skill A and B are demanded in the same ratio ($\frac{\nu}{\mu}$). However, the details of these settings are irrelevant to the main subject.

[9] Although there are few studies that support this assumption, the fact that primary education is provided by almost every government might be the strong evidence to support it. Haveman and Wolfe(1984) examine some educational externalities that might have been caused by primary education. Although some empirical studies report that

somewhat new explanation to so-called poverty trap in less developed countries. In the current model, as skill A is supposed to be a general-purpose skill, the proportion of people who spend a period of time learning it is decisive for the economic development in the country. Generally speaking, the more the skills are available in a country, the more the value to learn basic skill increases and hence, the more the country develops. Opposite hold in less developed countries to draw them into a poverty trap. There is, however, a different implication for the government policy depending on whether the government knows which skills (or industries that use the skills) are promising for economic development. If they are known, the government can shift the supply of more effective skills upwards by reforming the educational curriculum (that is, for example, by altering $\hat{f}(\alpha)$) or by implementing an appropriate subsidy policy. If they are not known, however, the economic growth of the country will be rather accidental. Countries in which ineffective skills are used in their main industries fail to achieve a high rate of growth. This view shows the merit of competition among governments, since it lets the government know which skills are more effective. In other words, the integration of regions or governments presents the risk of encouraging ineffective skills.

This paper also provides a new interpretation to the findings reported by Jovanovic and Nyarko(1996). Focusing on the value of understanding innate characteristics on the job training, they compared two models, namely Bandit model and Stepping stone model, both with 2 jobs (hence, two skills) that explains a job career path. Their interesting finding is that people tend to start their careers with difficult jobs and end up with simpler jobs in Bandit type model, which they conclude to be incompatible with reality. Instead they originally built and supported Stepping stone model which has a completely opposite implication from the Bandit model[10]. Although the model in this paper belongs to Bandit type model in their definition, our finding is that people may start their careers with simpler jobs if we loosen assumption in their Bandit type model and admit three skills to exist[11].

secondary education matters more for growth than primary education(see Krueger and Lindahl(2001)), as far as primary education provides people with information about their innate traits (i.e., the knowledge of which job they would be most suited for), the effect of secondary education on economic growth must be overestimated.

[10]See also Jovanovic and Nyarko(1995).

[11]In addition, Jovanovic and Nyarko(1996) assume the distribution of the two skills to be normal in Bandit type model, which is also crucial for their conclusion.

Although they do not mention the development stage of a country, their model, which has only two jobs available, can be considered to describe poor countries. In poor countries, where only a few jobs exist, people may well have a tendency to learn specialized skills for a particular job, since there would be no point in learning general-purpose skills to prepare for jobs that are not available in these countries. This behavior would prevent people from learning general-purpose skills, which results in a country remaining undeveloped.

3 2-Period, n-Skill, 1-Job Model

In this section, a theoretical model is presented to formalize the effect of economic development on the educational demand, by assuming two types of education. One is an education as an investment, which makes people more productive through the acquisition of information or knowledge concerning the usage of particular skills. This aspect of education is based on the Human Capital Theory originated by Becker (1964) and Schultz (1963). We denote this type of education as **type A education.** The other type of education is the one providing people with prospects for the future, which we described using a Bayesian probability in the previous section. To focus on this aspect of education, we introduce another type of education which we denote as **type B education**. We assume this type of education a priori, whose role is to reduce the uncertainty seen in **type A education**.

A good place to start is by considering an economic model with perfect foresight where people are certain about their educational outcomes. Then, by comparing it to an economy where they are uncertain, we make sure of the function of **type B education**. Finally, the model is extended so that the demand of **type B education** is solved endogenously in the model. The main result of the analysis is that the educational demand depends not only on what one can expect from a particular education but also on the degree of development in the region. While this model is somewhat simple mode discussed in the previous section, its advantage is to make us easier to grasp the effect of economic development on the demand of education.

The Economy with Perfect Foresight

Suppose a country where people (population is normalized to 1) can access n types of productivity-enhancing skills, and can decide whether to learn each of them. (Thus, this is how we define **type A education**.) We assume n to be continuous[12]. Individual can learn as many skills as he wants in exchange for an education cost C_i for each skill i ($i \in n$), and is assumed to maximize one's lifetime income. The education is completed before each of the individual gets a job. We assume for simplicity that there exists only one job in this economy, and that the acquisition of each skill contributes independently to increase one's productivity in the job. Educational outcomes are different among individuals (depending on the difference in abilities, aptitudes, or accidental reasons), where the degree that the skill i contributes to the increase of one's productivity in the job (which we denote as a_i) is supposed to be uniformly distributed in $[0, A_i]$. Thus, the lifetime earnings that each individual obtains are decided according to the outcomes on education. Let us see how an average productivity of a worker in this country can be described when people are certain about their educational outcomes in advance. Suppose that the credit market is perfect, and $0 \leq C_i \leq A_i$. Then, skill i is learned by $\frac{A_i - C_i}{A_i}$ of people whose average increase of productivity by the skill acquisition is $\frac{A_i - C_i}{2}$. Thus, the average productivity of a worker in this economy can be described as

$$\bar{I}^{PF} = \int_0^n \frac{(A_i - C_i)^2}{2A_i} di^{13}. \tag{38}$$

Note that **type B education** is not needed in this perfect foresight economy.

The Economy with Inperfect Foresight

We now consider a more realistic economy in which people do not know about their educational outcomes in advance. Thus, people can only realize their own values of a_i after they have spent an education cost C_i in learning skill i. In this case, the demand for skill i is 1 if $\frac{A_i}{2} \geq C_i$, and 0 if $\frac{A_i}{2} \leq C_i$. Thus, the average productivity of a worker in this economy can be described as

$$\bar{I}^{IF} \equiv \int_0^n \max\{\frac{A_i}{2} - C_i, 0\} di. \tag{39}$$

[12] This is a device for the analysis to be simple. Main results do not change even for a discrete n.

[13] The productivity of an individual without any education is assumed to be 0.

Note that the wage difference between perfect and imperfect foresight economy

$$\bar{I}^{PF} - \bar{I}^{IF} = \int_0^n \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\} di \tag{40}$$

is a social cost caused by an uncertainty of **type A education**.

We are now ready to define the other type of education, whose property is to reduce this uncertainty, consequently, making people more productive. Our interest is to understand the characteristics of the demand seen in this type of education, given the number of accessible skills n. Note that this type of education should be demanded, if at all, in the primary stage of one's education process, and that n can be regarded here to represent the degree of development in the country. Thus, our strategy is to examine the effect of economic development on the demand of education particularly in its early stages, by focusing on the relation between n and the demand of **type B education**. Suppose, after having **type B education**, one can choose to learn whichever skill one likes without any uncertainty if only one pays a fixed cost δ . That is, **type B education** is assumed to change the economy an individual faces from an imperfect foresight to the one with perfect foresight (In this way, **type B education** shows a prospective property to forecast one's outcome of future education.). Clearly, its demand is 1 if

$$\delta \leq \int_0^n \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\} di \tag{41}$$

is satisfied, in which case the differential between both side of the equation expresses the social benefit that **type B education** creates. It is easy to verify that there exists n^* for which the above equation holds for all $n \geq n^*$. Note that the impact of the number of accessible skills on the expected average income $(\Delta I/\Delta n)$ increases from $E_n(\max\{\frac{A_n}{2} - C_n, 0\})$ to $E_n(\frac{(A_n - C_n)^2}{2A_n})$ after $n = n^*$. This implies that in less developed countries where few skills or information are available, people have little incentive to have **type B education**. As seen in the previous section, this may make an innovation of new skill less effective, remaining the country poor (see Figure 6).

Determination of Primary Education Demand

The model is extended so that the demand of **type B education** is determined as an endogenous variable e (defined as the cost spent for **type**

B education) in the model. An individual who has invested e for **type B education** is supposed to realize one's ability of skill i (i.e. a_i) with a probability $P_i(e)$ $\left(\forall i, 0 \leq P_i(e) \leq 1, P_i'(e) > 0, P_i''(e) < 0\right)$. In this setting, the equation to give an expected average productivity of a worker in this economy is described as

$$\bar{I} = \int_0^n \left(P_i(e)\frac{(A_i - C_i)^2}{2A_i} + (1 - P_i(e)) \max\{\frac{A_i}{2} - C_i, 0\} \right) di - e. \tag{42}$$

An individual maximizes equation (42) on e to derive the first order condition,

$$\int_0^n P_i'(e) \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\} di = 1. \tag{43}$$

It is easy to check that the second order condition holds[14]. Next proposition characterizes the effect of economic development on the demand of **type B education** in this model.

Proposition 3.1.

$$\frac{de}{dn} > 0 \tag{44}$$

$$\frac{d^2\bar{I}}{dn^2} > 0 \tag{45}$$

Proof. From equation (43),

$$\frac{de}{dn} = -\frac{P_i'(e) \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\}}{\int_0^n P_i''(e) \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\} di} > 0. \tag{46}$$

On the other hand,

$$\begin{aligned} \frac{d\bar{I}}{dn} &= P_n(e(n))\frac{(A_n - C_n)^2}{2A_n} + (1 - P_n(e(n))) \max\{\frac{A_n}{2} - C_n, 0\} \\ &+ \left(\int_0^n P_i'(e) \min\{\frac{C_i^2}{2A_i}, \frac{(A_i - C_i)^2}{2A_i}\} di - 1 \right) e'(n) \\ &= P_n(e(n))\frac{(A_n - C_n)^2}{2A_n} + (1 - P_n(e(n))) \max\{\frac{A_n}{2} - C_n, 0\} \end{aligned}$$

[14]For simplicity, the existence of inner solution in equation (42) is assumed.

leads to

$$\frac{d^2\bar{I}}{dn^2} = P_n'(e(n)) \min\{\frac{C_n^2}{2A_n}, \frac{(A_n - C_n)^2}{2A_n}\} e'(n) > 0. \quad (47)$$

□

Equation (44) shows that as the variety of skills that exist in the country increases, the more the value of knowing one's innate ability increases, and therefore the more an individual invests for **type B education**. It also tells us that, since people have clearer prospects for **type A education**, the marginal increase of the variety of skills is more effective in the country with high n. Thus, equation (45) implies that an innovation or introduction of new skill is more effective in more developed countries.

4 Empirical Analysis

The purpose in this section is to examine how educational demand is affected by the variety of skills available in a region, or by other variables related to the possibilities which individuals have for their futures. Thus, we empirically analyze Proposition 3.1, using both Japanese prefectural data and international panel data. As a result, we find that the empirical results are consistent with our theory.

Data

Both the Japanese cross-section data and the international panel data are used to examine Proposition 3.1. Although it is very difficult to find the exact index that reflects the variety of skills available in a region, we use the number of industries in each prefecture for the Japanese analysis, and the proportion of employees in the non-agricultural sector for the international panel analysis. The number of industries (along with the number of enterprises in each industry) in each of Japanese prefecture is available every 5 years[15]. The proportion of employees in the non-agricultural sector for a wide range of countries is available from the World Development Indicators 2003 on CD-ROM (WDI). The proportion of consumption of supplementary education (i.e., education outside school) per student is used to represent

[15] All the statistics used for the Japanese cross-section analysis are collected from Statistics Bureau database (available from HP:www.stat.go.jp/english/).

education demand in the Japanese data set. Although it seems to be reasonable to include an expenditure for schooling, it is difficult to measure the exact level of expenditure that a household spends in each prefecture because schools are organized mostly by public expenditure (especially in the primary stages of education, which we are concerned with.). This is the reason why we have used the expenditure for supplementary education, as it seems to reflect the household's opinion of education directly. On the other hand, an enrollment rate is used to represent the index of educational demand for each country in the international panel analysis according to WDI and Global Education Digest 2003 published by UNESCO. The advantage of both the data is that we can obtain the level of educational demand for three stages of education: Pre-primary, primary, and secondary school education for Japanese data set and primary, secondary, and tertiary school education for the international data set.

Table 3 and Table 4 show correlations between the varieties of skills and the educational demand for the Japanese data (for the year 2000) and international panel data (for the year 2001) respectively. In Table 4, we also present the correlation coefficient, where an expenditure per student and a proportion of scientists (both available from WDI) are used instead of the enrollment rate and the proportion of non-agricultural workers . It is clear from the tables that the variables representing the variety of skills are more likely to be related to the educational demand of the earlier stages in both cases. This indicates that the **type B education** is more important in the early stages of education, since to be informed of one's talents or innate abilities is more valuable at a younger age. This is entirely consistent with what the theory predicts.

Results

Table 5 and Table 6 show the results where the educational demands (pre-primary for the Japanese data set and primary for the international data set) are regressed on the variety of skills and some other variables that relate to the possibilities or uncertainties individuals face.

Table 5 shows results from the cross-sectional data of Japanese prefectures. Since the statistics of educational demand have only been available since 1997[16](which we denote as data A) and the statistics of the number

[16]It is since 1997 that the expenditures for supplementary education have been classified

of industries is only available every 5 years, a simple OLS method is used for the Japanese cross-sectional data 2001. As well as the variety of skills, the unemployment rate is added to the regression, which is an index expected to reflect the uncertainties that children hold for their futures. Table 7 shows the correlation coefficient between unemployment rates and educational demands. Note that, like educational demand, the unemployment rate is more relevant to the earlier education in the Japanese cross-section data (On the other hand, we could no find the significant correlation between unemployment rates and enrollment rates for the international panel data.). The proportion of people moving to other prefectures every year (denoted as the moving rate), and the proportion of workers having a job in other prefectures (denoted as the rate of outside workers) is also added in the Japanese prefectural regression. Since the barrier between Japanese prefectures are low, the mobility of workers in each prefecture seems to be important for the future opportunities of the children, and hence for educational demand.[17] Though some explanation variables are not significant when using data A (second column in Table 5), the result improves when we use another data set (denoted as data B) instead (third column). Like data A, data B also shows the expenditure for supplementary education per student in each prefecture, available only for 1999[18]. The advantage of data B is that its samples cover wide range in each prefecture, contrary to data A whose samples are restricted mainly to large cities. Despite the disadvantage of the time gap between Data B and the data of the number of industries, the superiority in the samples seems to be why more solid results are obtained using data B. As far as we can believe that the propensity for the demand of supplementary education does not change much within a few years, the result obtained by using data B can be justified.

Table 6 shows the results of the panel data analysis (fixed effect model), by examining international panel data for the years 1970, 1980, 1990, and 2000[19]. Life expectancy at birth for each country is added to the regression since it increases the expected span that workers can engage in a job, and therefore increases the value of **type B education**. As seen in both Table 5 and Table 6, the explanation variables are roughly significant to educational demand,

into pre-primary, primary, and secondary education.

[17]Intra-national barriers seem to be much higher. In fact, we could not find significant relation between the intra-national labor mobility and educational demand.

[18]Also, available from Statistics Bureau database.

[19]Selection of the years is restricted by the availability of enrollment rates.

supporting equation (44) in Proposition 3.1 to hold. This indicates that people invest more on education if they face more possibilities or uncertainties in a region, since education (especially in its early stage) have an aspect that provides them with information concerning expected outcomes of their future education.

The marginal impact of the number of accessible skills on the average productivity (denoted as $\Delta GDP/\Delta n$) is also regressed on the variety of skill in both the Japanese cross-sectional data and the international panel data. ΔGDP and Δn are the increments of the real GDP per capita and the rate of non-agricultural worker respectively, between each periods for the international panel data (i.e., 1970-1980, 1980-1990, and 1990-2000). On the other hand, the real GDP per capita and the number of industries in each prefecture for the year 1991 is collected to produce ΔGDP and Δn for the Japanese cross-section data. Table 5 and Table 6 show that $\Delta GDP/\Delta n$ is positively related to the variety of skills in the region in both cases, indicating that equation (45) in Proposition 3.1 also holds. This means that innovation or introduction of a new skill contributes more to increase one's productivity if more skills are already available in the region. Since the demand for **type B education** is larger, and hence people are more informed of their innate traits and abilities in such regions, people are more adaptable to a new technology. These empirical results support our hypothesis that this aspect of **type B education** is quite important for educational demand especially in its early stages.

The Role of Government

This paper suggests that, contrary to people in advanced countries, people in less developed countries where only few skills are available have less incentive to acquire general skills. Instead, they seem to be favor of learning rather specific skills. Since general skills (or basic skills) are believed to be very important for the development of countries, this fact indicates that less developed countries have possibilities to be caught by poverty traps. In fact, this implication is consistent to Psacharopoulos(1985) which broadly compares returns to education among different countries and different grades of education. Table 8 is an exert from empirical results in Psacharopoulos(1985). Notice that it is in the primary stage of education that we can find large gap of social return to education between advanced countries and less developed countries, which is consistent with our results. This suggest us that the government in less developed countries have more incentives to enforce the program of compulsory education.

5 Conclusion

Two types of models are presented in this paper to analyze the demand of education theoretically. The role of education that it provides people with prospects for the future, is expressed in respective ways. People who are uncertain of their educational outcomes are sorted into distinct skill acquisition paths due to this property in our first model. In the second model, we have shown that the variety of skills or possibilities one face is important for the demand of education in its early stage. These models have made it possible for us to understand the effect of economic development on the demand of education. In the latter part of this paper, this theoretical prediction has proved to be consistent with empirical results examined by both Japanese cross-section data and international panel data particularly in the earlier stages of education. Our framework also implies that people in a region where few skills are available have a tendency not to learn much, as they have less incentive to obtain information about their innate abilities, resulting in the region remaining poor.

6 Appendix

Functions used for the numerical analysis. (Section 2)

$$f(\alpha,\beta,\gamma)=\begin{cases} u(\alpha)v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma)=(0,0) \\ (1-u(\alpha))\,v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma)=(M_B,0) \\ u(\alpha)\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma)=(0,M_C) \\ (1-u(\alpha))\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma)=(M_B,M_C) \\ 0 & \text{elsewhere} \end{cases}, \tag{48}$$

$$\hat{f}(\alpha)=\frac{\phi(\alpha)-\phi(0)}{\int_0^{M_A}\phi(\alpha)d\alpha-M_A\phi(0)} \tag{49}$$

$\phi(\alpha)$ is the normal distribution with a mean m and standard deviation σ

$$u(\alpha) = \begin{cases} -\frac{1}{b}\alpha+1 & (\alpha\le b) \\ 0 & (\alpha\ge b) \end{cases} \tag{50}$$

$$v(\alpha) = \begin{cases} 1 & (\alpha\le \delta c) \\ -\frac{1}{(1-\delta^2)c^2}\alpha^2+\frac{1}{1-\delta^2} & (\delta c\le\alpha\le c) \\ 0 & (\alpha\ge c) \end{cases}, \tag{51}$$

$(0\le b\le \hat{p}_B M_B,\ 0\le c\le \hat{p}_C M_C,\ 0\le\delta\le 1)$

Values of parameters used for the numerical analysis.

$M_A=8$	$b=1$	$m=4$	$\nu/\mu=0.08$
$M_B=4$	$c=10$	$\sigma=2$	$(1-\mu-\nu)/\mu=0.25$
$M_C=20$	$\delta=0.5$	$L=1$	

Descriptive statistics used in the empirical analysis. (Section 4)

Japanese prefectural data

	Average	Variance	Max	Min	obs.
Supplementary education (%)					
(Data A: 2001)[a]					
Pre-primary	0.72	0.15	1.68	0.14	47
Primary	4.16	2.41	8.92	2.32	47
Secondary	2.98	3.42	8.56	0.80	47
(Data B:1999)[b]					
Pre-primary	0.59	0.072	1.48	0.11	47
Primary	4.27	2.02	6.73	1.29	47
Secondary	2.98	1.08	4.74	0.51	47
No. of industries (2001)	451	334	485	409	47
No. of industries(1996)	447	342	482	410	47
No. of industries(1986)	430	286	470	394	47
Moving rate (2001)	2.06	0.14	3.15	1.27	47
Outside workers (2001)	5.29	60.3	30.9	0.1	47
Unemployment rate (2001)	4.85	1.04	8.4	3.2	47
GDP per capita (1996)	2857	1.67×10^5	4180	2096	47
GDP per capita (1986)	2077	1.05×10^5	3395	1620	47

Source: Statistics Bureau, Ministry of Public Management.

[a] Family Income and Expenditure Survey.

[b] National Survey of Family Income and Expenditure.

International data

		Average	Variance	Max	Min	obs.
Enrollment rate (%)						
	Primary	57.0	1.1×10^3	99.5	0.4	97
1970	Secondary	17.8	308	69.8	0.3	99
	Tertiary	3.3	18.0	21.5	0.0	101
	Primary	62.3	948	100	7.3	101
1980	Secondary	24.9	419	92.7	0.7	101
	Tertiary	5.8	44.4	37.4	0.1	101
	Primary	69.7	738	100	11.1	104
1990	Secondary	33.2	490	89.6	1.7	104
	Tertiary	8.5	69.9	45.2	0.1	104
	Primary	74.6	558	100	13.6	104
2000	Secondary	38.3	505	89.7	2.9	104
	Tertiary	11.5	100	53	0.2	104
Expenditure per student (US$)						
	Primary	14.7	8.4	45.8	2.1	77
2000	Secondary	23.0	15.9	81.0	4.7	83
	Tertiary	105	221	1.4×10^3	5.5	86
Life expectancy at birth						
1970		58.4	129	74.6	34.4	101
1980		62.2	114	76.1	35.4	103
1990		65.0	114	78.8	35.2	104
2000		65.8	161	80.7	38.0	104
Proportion of non-agriculture worker (%)						
1970		50.0	75.3	98.2	5.6	146
1980		59.4	866	98.8	7.2	115
1990		61.6	784	100	6.4	147
2000		85.3	250	99.7	23.4	87
GDP per capita (US$, constant 1995)						
1970		4.4×10^3	6.9×10^3	3.5×10^3	120	102
1980		5.7×10^3	8.4×10^3	4.0×10^3	147	114
1990		5.6×10^3	9.0×10^3	4.6×10^3	100	139
2000		6.4×10^3	1.1×10^4	4.7×10^3	116	140
Number of scientists (per million)						
2000		1.2×10^3	1.2×10^3	5.0×10^3	2.2	86

Source: World Development Indicators, Global Education Digest (UNESCO).

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Table 1: State of the economy.

The equilibrium skill price.		
Skill B ($\hat{p}_B^*$)	1.07	
Skill C ($\hat{p}_C^*$)	2.67	
Proportion within labor force population.		
Occupation A	79.3%	
Occupation B	6.1%	
Occupation C	1.5%	
Learning B	6.5%	
Learning C	6.5%	
Average wage.		
	Market-observed	Latent
Occupation A	4.1	4.0
Occupation B	4.3	4.2
Occupation C	53	2.1

Table 2: Expected lifetime income in 2-skill economy and in 3-skill economy.

	2-skill economy	3-skill economy
$E(I_A^{EQ})^{poor}$	8.16	
$E(I_B^{DEV})^{poor}$	8.87	
$E(I_A)$		8.72
$E(I_B^{DEV})$		8.50
$E(I_C^{DEV})$		8.61

Table 3: The correlation coefficient between the demand for supplementary education and the number of industries for Japanese 47 prefectures 2001.

Correlation Demand for supplementary education	No. of industries
Pre-Primary	0.332
Primary	0.291
Secondary	0.170

Source: Statistics Bureau. http://www.stat.go.jp/english/index.htm

Table 4: The correlation coefficient between educational damand and the variety of skills for international data set 2000.

Correlation	education level	Proportion of non-agricultural workers	No. of Scientists (per million)
Enrollment rate[a]	Primary	0.757 (77)	0.643 (81)
	Secondary	0.584 (72)	0.424 (79)
	Tertiary	0.557 (58)	0.622 (61)
Expenditure per Student[b]	Primary	0.247 (47)	0.619 (47)
	Secondary	0.104 (52)	0.270 (50)
	Tertiary	-0.148 (53)	-0.268 (57)

Source: World Development Indicators 2003 (World Bank) and Global Education Digest 2003 (UNESCO).

Note: The number of observations are in parenthesis.

[a] The net enrollment rate for primary education, and the gross enrollment for secondary and tertiary education.

[b] The public current spending on education divided by the total number of students by level, as a percentage of GDP per capita.

Table 5: Estimation Results for the Effects on Education Demand for 47 Japanese prefectures. (OLS method using data A (2001) and data B (1999).)

	Expenditure for supplementary education(%) (pre-primary education)		$\Delta(GDP)/\Delta n$
	Data A	Data B	
No. of industries	4.66×10^{-3}	3.77×10^{-3}	6.08
	(1.69*)	(2.33**)	(2.44**)
Moving rate	0.185	0.181	
	(1.14)	(1.98**)	
Rate of outside workers	4.66×10^{-3}	5.68×10^{-3}	
	(0.537)	(1.26)	
Unemployment rate	0.082	0.078	
	(1.94**)	(3.27**)	
F-statistic	4.14	12.1	5.96
adj.R2	0.283	0.535	0.122
obs.	47	47	45[a]

t-values are in parenthesis.

** Significant at 1% level. * Significant at 5% level.

[a] Two samples are missed because Δn is 0.

Table 6: Estimation Results for the Effects on Education Demand for internationl panel analysis 1970,1980,1990,and 2000 (fixed effect model).

	Enrollment Rate (primary education)	$\Delta(GDP)/\Delta n$
Rate of non-agricultural workers	0.549 (8.89**)	1.99×10^{-3} (2.15*)
Life Expectancy	0.703 (5.77**)	
Hausman statistics	29.7**	5.27**
adj.R2	0.958	0.347
obs.	324	214

t-values are in parenthesis.

** Significant at 1% level. * Significant at 5% level.

Table 7: The correlation coefficient between educational demand and unemployment rate.

	Correltaion between unemploment rate	
Educational demand	Japanese data	International data
Pre-primary	0.409	
Primary	0.180	0.112
Secondary	0.102	-0.107
Tertiary		-0.142

Source: Statistics Bureau (http://www.stat.go.jp/english/index.htm) and World Development Indicators 2003 (World Bank).

Table 8: Social Return to Educatoin(%)

	Social Return to Education		
	Primary	Secondary	Post-secondary
Morroco	50.5	10.0	13.0
Ethiopia	20.3	18.7	9.7
Kenya	21.7	19.2	8.8
India	29.3	13.7	10.8
Chinese tie-pei	27.0	12.3	17.7
Peru	41.4	3.3	16.1
Mexico	25.0	17.0	23.0
Greece	16.5	5.5	4.5
Israel	16.5	6.9	6.6
Spain	17.2	8.6	12.8
Singapole	6.6	17.6	14.1
France	–	10.1	10.9
Japan	9.6	8.6	6.9
Great Britain	–	9.0	7.0
Unites States	–	10.7	10.9

Excerpt from Psacharopoulos(1985)

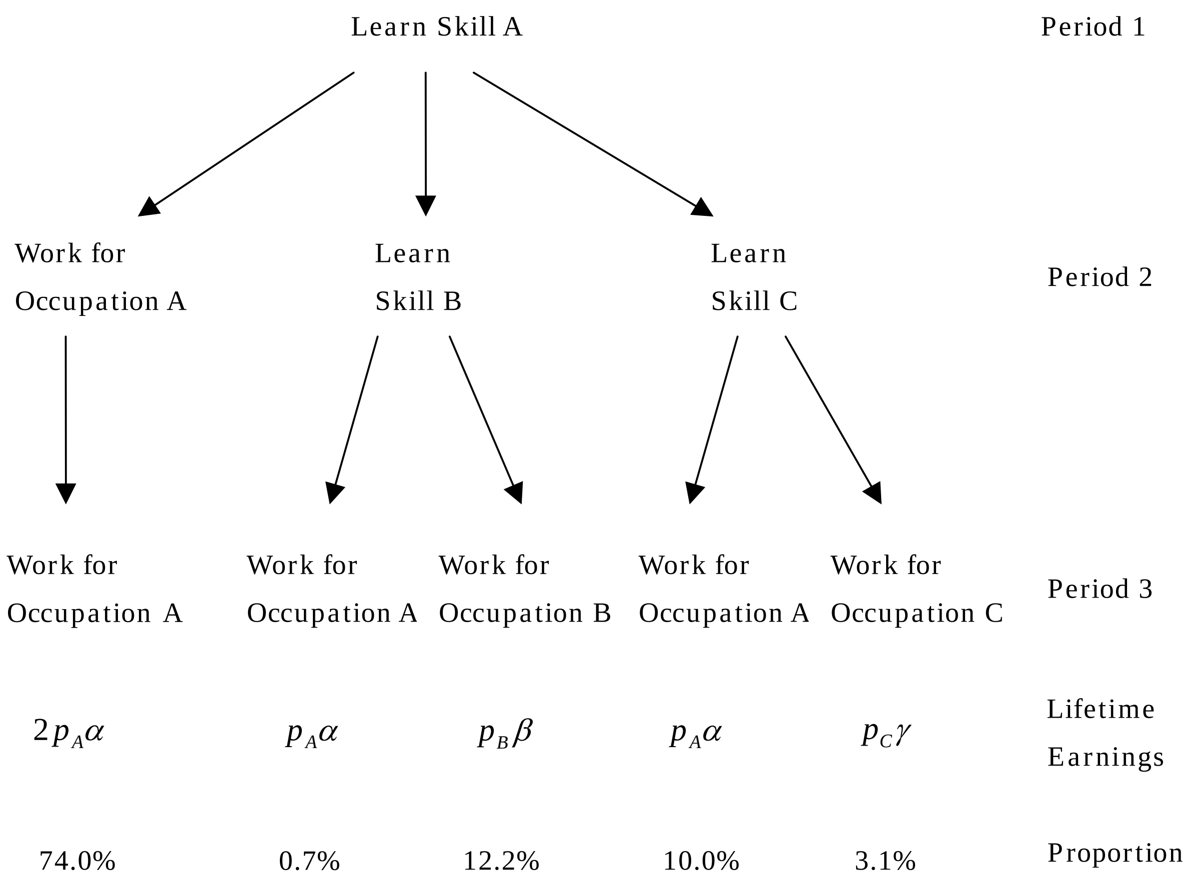


Figure 1. Skill Acquisition Paths.
(The proportional rate in each skill acquisition path is based on the numerical analysis in Section 3.)

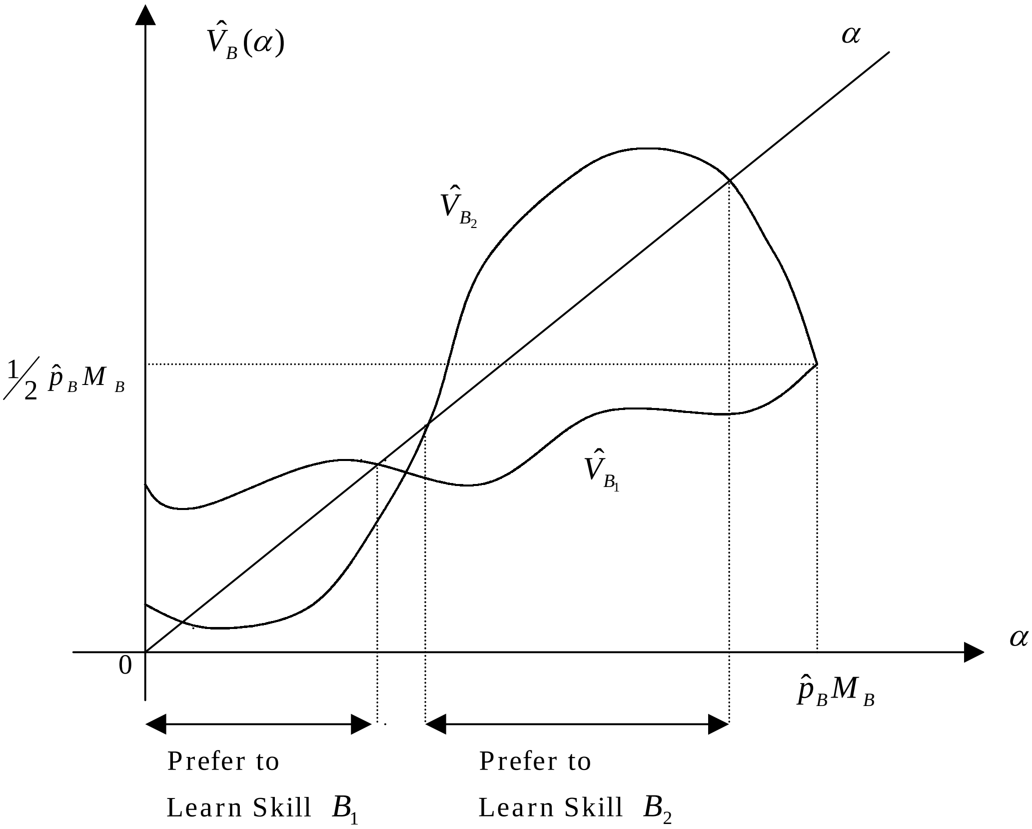


Figure 2. $\hat{V}_B(\alpha)$ and the 45° line.

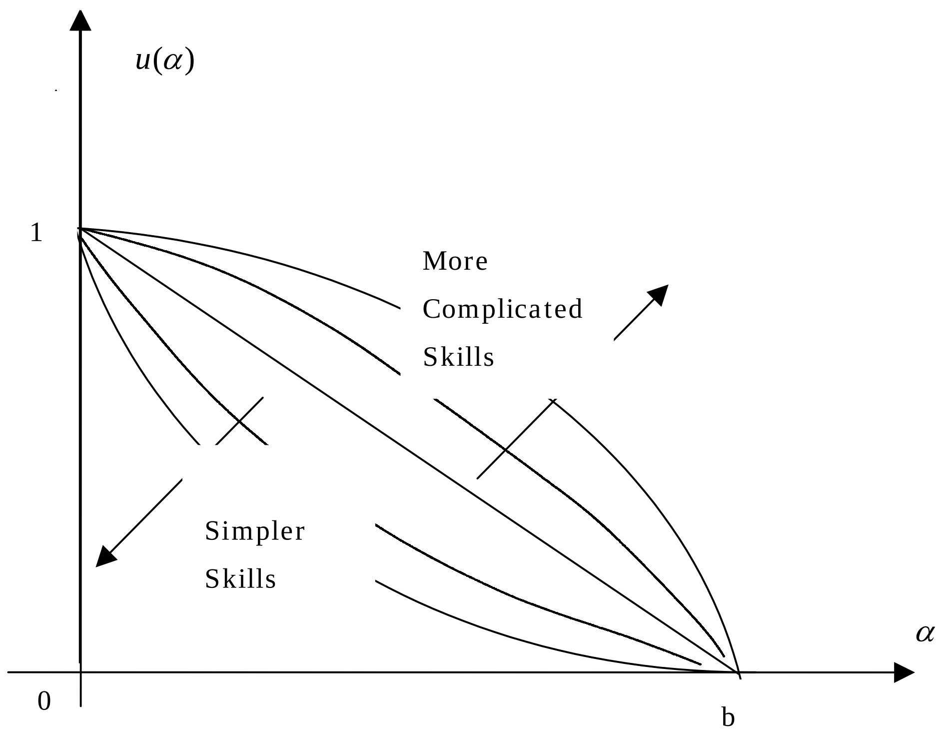
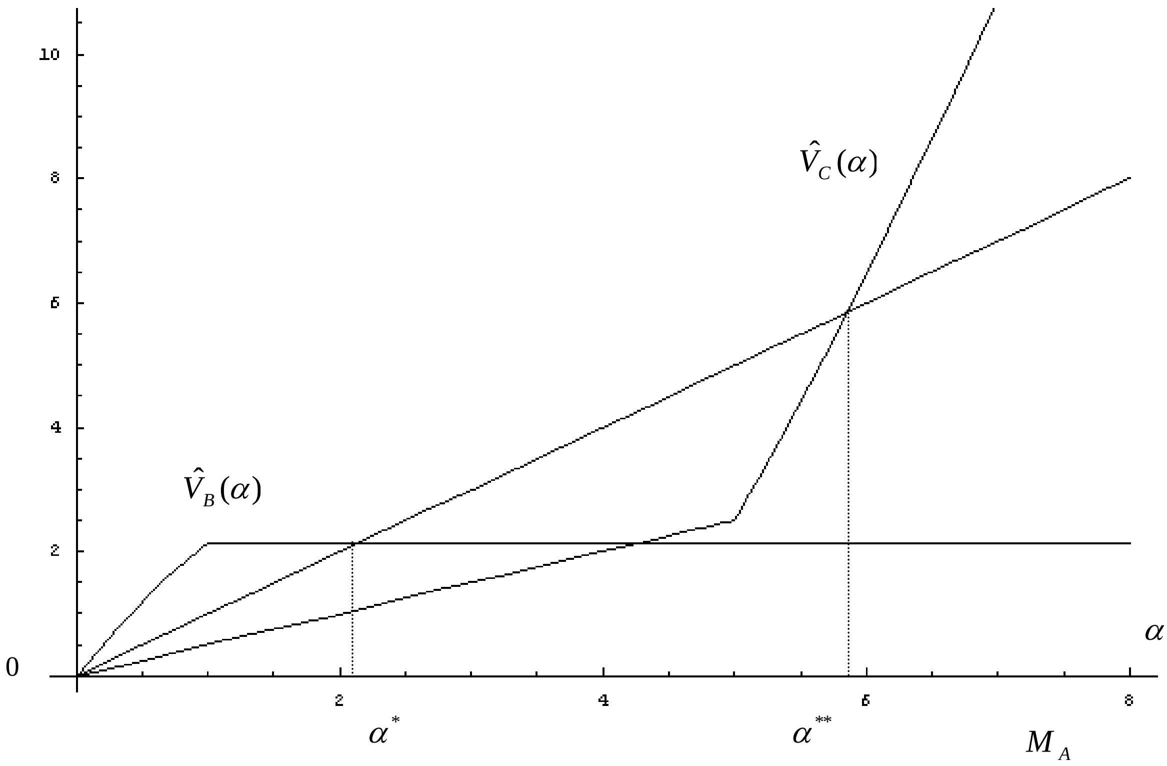


Figure 3. Characteristics of skill B.



$\alpha^* = 2.08 \qquad \alpha^{**} = 5.85$

Figure 4. $\hat{V}_B(\alpha)$, $\hat{V}_C(\alpha)$, and the 45° line in the equilibrium.

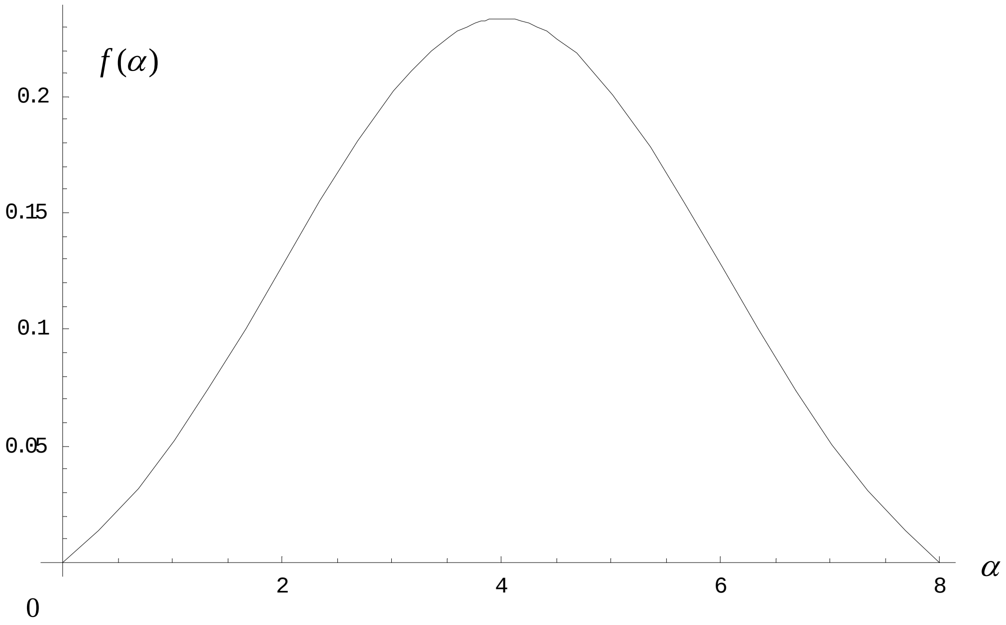


Figure 5.1. The distribution of basic skill ability.

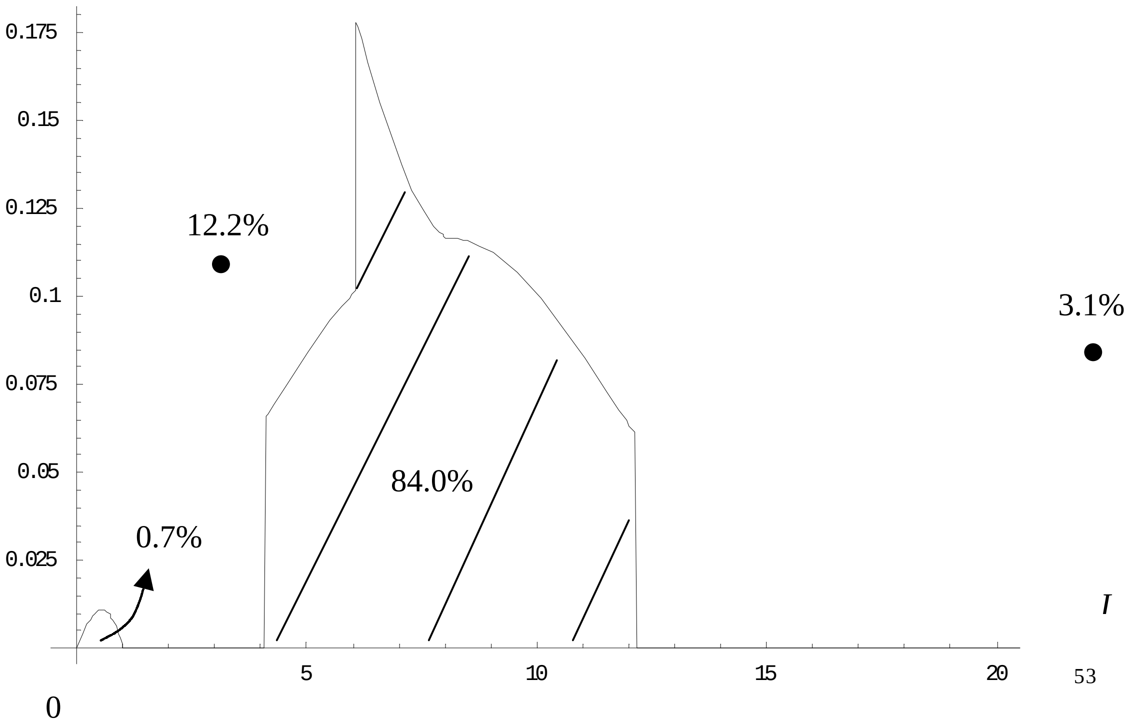


Figure 5.2 The distribution of lifetime earnings.

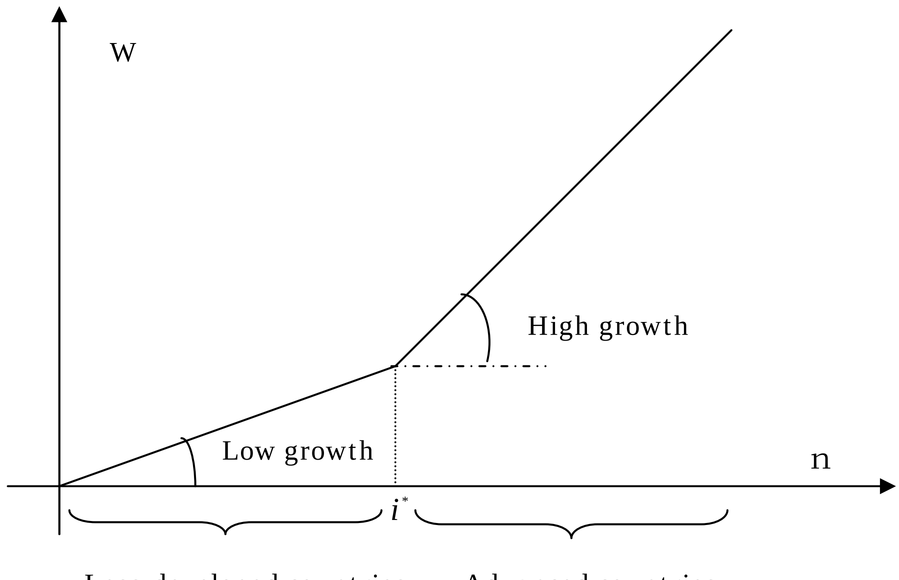


Figure 6: The variety of industries and the average wage.