

Discussion Paper No. 013

Cooperative Licensing in the Bertrand Competition  
with Differentiated Commodities

Naoki Watanabe

February, 2004

21COE  
Interfaces for Advanced Economic Analysis  
Kyoto University

# Cooperative Licensing in the Bertrand Competition with Differentiated Commodities

Naoki Watanabe\*

February 2004

## Abstract

We formalize a cooperative licensing game in the Bertrand triopoly with differentiated commodities, where an outside patent holder wishes to license to firms its new technology that reduces the unit cost of production. Applying a new way of deriving  $v$ -function from games in strategic form, we consider stable coalition structures with distribution rules that form endogenously in the process of writing agreements. It is most profitable for the patent holder to license its new technology to two firms if the commodities are not so substitutive, or to only one firm if they are quite substitutive.

*Keywords* : licensing, process innovation, product differentiation  
coalition structure, distribution rules, binding agreement

*JEL Classification Numbers* : D45, D43, C71

---

\*postdoctoral fellow (COE), Graduate School of Economics, Kyoto University, Sakyo, Kyoto 606-8501, Japan. email: [watanabe@kier.kyoto-u.ac.jp](mailto:watanabe@kier.kyoto-u.ac.jp) This research was partially supported by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Grant-in-Aid for 21 Century COE Program. The author wishes to thank Prof. Akira Okada and Prof. Haruo Imai for their intellectual guidance. He is grateful also to Prof. Takanori Ida.

# 1 Introduction

Patent licensing problems have ever been studied mainly in the Cournot competition, whereas the literature in the Bertrand competition is extremely limited. Kamien and Tauman (1986) and Kamien, Oren and Tauman (1987) showed a result, but it was quite simple: regardless of licensing schemes, an outside patent holder licenses exclusively to one firm its new cost-reducing technology, and every non-licensee firm is to be expelled from the industry at the equilibrium price, because of the homogeneity of commodities.

Muto (1993) studied the optimal licensing scheme, introducing a patent holder into the model analyzed by Singh and Vives (1984). As shown in Kamien and Tauman (1986), it is never optimal for the patent holder to license by means of the royalty in the Cournot competition, but he showed: for an outside patent holder licensing by means of the royalty may be superior to licensing by means of a fixed fee or auction in the Bertrand duopoly with differentiated commodities.

On the other hand, it might be possible that a patent holder licenses the new technology to a limited number of firms through negotiations with them on (1) how to cooperate in opposition to (collusive) pricing by the non-licensee(s) and (2) how to split the total sum of profits realized by licensees. We investigate this cooperative aspect of the patent licensing problems, extending duopoly models studied by Muto to triopoly.

To formalize such a situation, we consider coalition structures that form endogenously in the process of writing agreements: players are partitioned into some coalitions, each coalition has a distribution rule about how to divide the joint profit of the coalition, and the action of each firm in each coalition should be compatible with the firm's payoff maximization in some sense. Given a coalition structure with distribution rules, if every firm in every coalition agrees "individually" upon the action required to take in the coalition, the list of such actions is regarded as a "binding agreement".

In the standard analysis of cooperation, agreements in coalitions are considered as contracts and studied with games in coalitional form, where the worth of each coalition is given a priori by a v-function. Our approach is, however, based on non-cooperative behaviors of players, and then it is necessary to represent strategic-form games in coalitional form in some way. In this paper, we apply a new way of deriving a v-function suggested by Watanabe and Tauman (2003).

It would be difficult to apply the classical way of deriving v-function suggested by von Neumann and Morgenstern (1944) and Aumann (1959) to the situation in our question: if a coalition  $S$  of players broke off an original coalition structure cooperating only within members in  $S$ , all the other players not in  $S$  would minimize their payoffs as a punishment, according to the classical way. It might not be, however, in the best interests of the players to minimize the payoffs of the members of the deviating coalition. In general, it would not be compatible with the payoff maximization of each firm taking action in each coalition. We hence introduce a self-enforceability of punishments into the derivation of v-function. The key feature is coalition formations with spin-offs of several subcoalitions in each coalition.

Ray and Vohra (1997) studied a concept that is similar to ours, which is recursively defined without deriving v-function because of the above problem of the classical derivation. They considered the “internal deviation” of a coalition  $S$ , where a new coalition structure  $P'$  can be formed only through the disintegration of coalitions existing under the original coalition structure  $P$ . Denote by  $(P', s')$  a pair of a new coalition structure  $P'$  and a list  $s'$  of players' strategies called a binding agreement under  $P'$  appearing after the deviation of  $S$ . The pair  $(P', s')$  is regarded as credible, if any re-merging of the players in each original coalition except members in deviating coalition  $S$  is blocked by  $(P', s')$ . The list  $s^*$  of players' actions under a coalition structure  $P^*$  is called an “equilibrium” binding agreement, if  $s^*$  is a binding agreement under  $P^*$  and there is no credible  $(P', s')$  that blocks  $(P^*, s^*)$ .

Avoiding the representation problem of worths of coalitions in strategic-form games, Ray and Vohra formalized a new framework to analyze coalition structures in a sophisticated way. Because of the recursive definition, however, the computation process tends to be much more complicated as the number of players increases, even in the symmetric Cournot competition.

Our solution concept captures the blocking process they formalized in the new derivation of v-function, and so it is quite similar to what they called an equilibrium binding agreement. In that sense, our solution concept can be considered as a simplification of their argument. It is most profitable for the patent holder to license its new technology to two firms if the commodities are not so substitutive, or to only one firm if they are quite substitutive.

The next section formalizes the cooperative patent licensing game in the Bertrand competition with differentiated commodities, defining the new way of deriving a v-function and solution concept called a binding agreement. Section 3 shows our results after providing some lemmata. Proofs are given mainly in Appendix. Section 4 closes this paper with a brief discussion.

## 2 The Model

### Cooperative Patent Licensing Games

There are three firms in this industry. Each firm  $i$  produces  $q_i (\geq 0)$  units of a differentiated good with the unit price  $p_i (\geq 0)$  and the unit cost  $c_i (> 0)$  of production, where  $i \in N = \{1, 2, 3\}$ .  $N$  is the set of players and  $|N| = n = 3$ . Consumers are identical and the representative consumer maximizes  $U(q_1, q_2, q_3) - p_1q_1 - p_2q_2 - p_3q_3$ , where

$$U(q_1, q_2, q_3) = \alpha(q_1 + q_2 + q_3) - \frac{\beta}{2}(q_1^2 + q_2^2 + q_3^2 + 2\gamma(q_1q_2 + q_2q_3 + q_1q_3)),$$

with  $\alpha > 0$ ,  $\beta = 1$  and  $0 < \gamma < 1/(n+1) = 1/4$ .  $\beta$  is normalized. The first order condition with respect to  $q_i$  is  $\alpha - q_i - \gamma q_j - \gamma q_k = p_i$ , and so each

demand  $q_i(p) = q_i(p_i, p_j, p_k)$  for firm  $i = N$  is given by

$$\begin{aligned} q_i(p) = \max(y_i(3) = \frac{\alpha(1-\gamma) - (1-\gamma)p_i + \gamma p_j + \gamma p_k}{1-\gamma+1-\gamma^2}, 0) & \quad \text{if } q_j > 0 \text{ and } q_k > 0 \\ \max(y_i(2) = \frac{\alpha(1-\gamma) - p_i + \gamma p_i}{1-\gamma^2}, 0) & \quad \text{if } q_j > 0 \text{ and } q_k = 0 \\ \max(y_i(1) = \alpha - p_i, 0) & \quad \text{if } q_j = q_k = 0, \end{aligned}$$

The numbers in brackets of  $y_i$  indicate how many firms supply the commodities to the industry. Commodities produced by firms are considered as substitutes, since  $\gamma > 0$ . Simpler expressions are as follows:

$$\begin{aligned} y_i(3) &= a(3) - d(3)p_i + e(3)(p_j + p_k) \\ y_i(2) &= a(2) - d(2)p_i + e(2)p_j \\ y_i(1) &= a(1) - d(1)p_i \end{aligned}$$

where  $a(3) = \alpha(1-\gamma)/(1-\gamma+1-\gamma^2)$ ,  $d(3) = (1-\gamma)/(1-\gamma+1-\gamma^2)$  and  $e(3) = \gamma/(1-\gamma+1-\gamma^2)$ , where  $a(2) = \alpha(1-\gamma)/(1-\gamma^2)$ ,  $d(2) = 1/(1-\gamma^2)$  and  $e(2) = \gamma/(1-\gamma^2)$ , and where  $a(1) = \alpha$  and  $d(1) = 1$ . We omit the number of firms with  $q_i > 0$  unless it is confusing.

Note that  $y_i = a - bp_i + e(p_i + p_j + p_k)$  with  $d = b - e$ . We can decompose the changes in  $y_i$  into two parts, the ‘‘own price effect’’ and the ‘‘price index effect’’ in the industry. It seems reasonable to assume that  $0 < \gamma < 1/4$ , i.e.  $b > 4e$ , meaning that the own price effect is sufficiently bigger than the price index effect when each firm raises the price by one unit.

$\gamma$  might be different among any different pairs of commodities  $i$  and  $j$ . We consider the symmetric case ( $\gamma_{ij} = \gamma$ ) to show our idea with simpler illustrations<sup>1</sup>. Hence, let  $c_i = c$  with  $\alpha - c > 0$  assumed<sup>2</sup>.

<sup>1</sup>Singh and Vives (1984) assumed  $|\gamma| < 1$  and interpreted it as the rate of substitution: commodities produced by firm  $i$  and firm  $j$  is substitutes (complements) if  $\gamma_{ij} > 0$  ( $< 0$ ). However, it might not be best, since if we extend their model to  $n$ -firm case, we should ensure that  $e(n) \rightarrow 0$  as  $n \rightarrow \infty$ , which leads to the *monopolistic competition*.  $|\gamma| < 1/(n+1)$  is a sufficient condition for it. It could be that  $e < 0$  if  $\gamma < 0$ .

<sup>2</sup>This condition comes from the social welfare issue: since  $p_i = \alpha - q_i(1)$ , the total sum of the consumer surplus and producer surplus in firm  $i$ 's market is measured by  $\alpha - q_i(1) - c$ , which should be positive if  $q_i(1) > 0$  is socially desirable.

We introduce into the symmetric triopoly described above a patent holder of a new technology which reduces the unit cost of production from  $c$  to  $c - \epsilon (> 0)$ . The individual profit of firm  $i$  is  $u_i(p) = q_i(p)(p_i - C_i)$ , where  $C_i = c - \epsilon$  if firm  $i$  is a licensee and  $C_i = c$  if  $i$  is a non-licensee. The patent holder takes no action at the market (outside patent holder), but would like to share some amount of profits of licensee firms in reward for licensing the new technology. The set of players in the cooperative licensing game is  $N' = \{0\} \cup N$ , where player 0 stands for the patent holder.

The patent licensing game with fixed fees is as follows. The patent holder first announces the prices of the new technology to firms, and each firm next decides simultaneously and independently of the other firms whether or not to purchase it with each price shown by the patent holder. Finally they compete in prices, knowing that which firms are licensed or not. We hereafter re-formalize this situation as a cooperative game with side payments.

Consider a partition  $P = [S^0, S^1, \dots, S^M]$  of  $N'$ . Each nonempty subset  $S^m \subset N'$  is called a *coalition* of players.  $S^0$  is the coalition of the patent holder and all the licensee firms, and each coalition  $S^m$  with  $m \neq 0$  is of non-licensee firms only. In each coalition  $S^m$ ,  $m = 0, \dots, M$ , there can be a subpartition  $\tilde{S}^m = \{S^{1(m)}, \dots, S^{l(m)}, \dots, S^{L(m)}\}$ . Each nonempty subset  $S^{l(m)} \subset S^m$  is called a *subcoalition* of firms.  $L(m)$  is the number of subcoalitions spun off in  $S^m$ . Denote by  $\tilde{P} = [\tilde{S}^0, \tilde{S}^1, \dots, \tilde{S}^M]$  a *coalition structure*<sup>3</sup>.  $M$  is the number of coalitions in  $\tilde{P}$ .

Given a coalition structure  $\tilde{P}$ , each subcoalition  $S^{l(m)}$  in coalition  $\tilde{S}^m$  competes in prices not only with all the “opponent” subcoalitions outside  $\tilde{S}^m$ , but also with the other “colleague” subcoalitions in  $\tilde{S}^m$ , while firms in the same subcoalition  $S^{l(m)}$  cooperate with one another to maximize the total sum of individual profits  $u_{S^{l(m)}}(p) = \sum_{i \in S^{l(m)}} u_i(p)$  in the subcoalition.  $u_i(p) = q_i(p)(p_i - C_i)$ , where  $C_i = c - \epsilon$  if  $i \in S^0$ , and  $C_i = c$  if  $i \notin S^0$ .

---

<sup>3</sup>Our definition of coalition structure is different from the usual one in the literature.

**Definition 1** We say that a strategy profile  $\bar{p} = (\bar{p}_i)_{i \in N}$  of prices satisfies the **best response property** under a given coalition structure  $\tilde{P}$  if for every subcoalition  $S^{l(m)}$  in every coalition  $\tilde{S}^m \in \tilde{P}$  there is no  $p_{S^{l(m)}} = (p_i)_{i \in S^{l(m)}}$  such that  $u_{S^{l(m)}}(p_{S^{l(m)}}, \bar{p}_{-S^{l(m)}}) > u_{S^{l(m)}}(\bar{p})$ . Denote by  $\beta(\tilde{P})$  the set of such strategies.

We restrict our attention to pure strategies. The strategy profile  $\bar{p} \in \beta(\tilde{P})$  is then the solution of the following:

$$\begin{aligned} \max_{(p_i)_{i \in S^{l(0)}}} \sum_{i \in S^{l(0)}} q_i(p) (p_i - c + \epsilon) & \quad \forall S^{l(0)} \in \tilde{S}^0 \\ \max_{(p_i)_{i \in S^{l(m)}}} \sum_{i \in S^{l(m)}} q_i(p) (p_i - c) & \quad \forall S^{l(m)} \in \tilde{S}^m, m \neq 0 \end{aligned}$$

Assume that  $p_i$  is a continuous variable for all  $i \in N$ .  $\beta(\tilde{P})$  is then nonempty.

Denote by  $\pi(S^m)$  the set of possible subcoalitions in  $S^m$  and denote by  $\tilde{\pi}(N' \setminus S^m)$  the set of possible coalition structures defined over  $N' \setminus S^m$ , respectively. Let  $\pi \in \pi(S^m)$  and  $\tilde{\pi} \in \tilde{\pi}(N' \setminus S^m)$ . The equilibrium joint profit of each coalition  $S^m$  under  $\tilde{P} = [\tilde{\pi}, \pi]$  is naturally defined as the total sum of equilibrium profits  $\sum_{S^{l(m)} \in \pi} u_{S^{l(m)}}(\bar{p})$  of its subcoalitions.

**Definition 2** Let  $v$  be a v-function that assigns to each  $S^m \subset N'$  a real number  $v(S^m)$ . The **guaranteed worth**  $v(S^m)$  of coalition  $S^m \neq \{0\}$  is defined as

$$v(S^m) = \min_{\tilde{\pi} \in \tilde{\pi}(N' \setminus S^m)} \max_{\pi \in \pi(S^m)} g_{S^m}(\tilde{\pi}, \pi),$$

where  $g_{S^m}(\tilde{\pi}, \pi) = \min_{\bar{p} \in \beta(\tilde{P})} \sum_{S^{l(m)} \in \pi} u_{S^{l(m)}}(\bar{p})$  with  $\tilde{P} = [\tilde{\pi}, \pi]$  given, and  $v(\{0\}) = 0$ . Assume that  $v(\emptyset) = 0$ .

### The Solution Concept

Given a coalition structure  $\tilde{P}$ , the members of each coalition  $\tilde{S}^m \in \tilde{P}$  divide the equilibrium joint profit of  $\tilde{S}^m$  among them and determine a vector  $x_{S^m} = (x_i)_{i \in S^m} \in R^{|S^m|}$  of payoffs, according to the distribution rule in  $S^m$ . Different coalitions are allowed to have different distribution rules.

Let  $d(\tilde{S}^m, \tilde{P})$  denote the distribution rule in coalition  $\tilde{S}^m$  under a given coalition structure  $\tilde{P} = [\tilde{S}^0, \dots, \tilde{S}^M]$  and  $D(\tilde{P}) = (d(\tilde{S}^0, \tilde{P}), \dots, d(\tilde{S}^M, \tilde{P}))$  denote a list of distribution rules under  $\tilde{P}$ .  $d(S)$  is the distribution rule in a coalition  $S$  deviating from an original coalition structure  $\tilde{P}$ . The v-function  $v_{S^m}$  for the negotiation over  $x_{S^m}$  within  $S^m$  is a natural restriction on  $v$  by  $S^m$ :  $v_{S^m}(S^m) = \sum_{i \in S^m} u_i(\bar{p})$  and  $v_{S^m}(S) = v(S)$  for any  $S \subset S^m$ .

Any distribution rules require (1) *individual rationality*:  $x_i \geq v(\{i\})$  for all  $i \in \tilde{S}^m$  and  $S$ , (2) *efficiency*:  $\sum_{i \in S^m} x_i = v_{S^m}(S^m)$  for each  $S^m$  and  $\sum_{i \in S} x_i = v(S)$  and (3) *symmetry*: for any coalition  $\tilde{S}^m$  under any  $\tilde{P}$  and for any deviating coalition  $S$ , identical firms in  $\tilde{S}^m$  and  $S$  should be given the same amount of payoffs<sup>4</sup>. Given the distribution rules under any coalition structures, the cooperative licensing game is defined as  $(N', v)$  with  $D(\tilde{P})$ .

**Definition 3** Given a coalition structure  $\tilde{P}$  and a list  $D(\tilde{P})$  of distribution rules under  $\tilde{P}$ , a strategy profile  $\bar{p} \in \beta(\tilde{P})$  is a *binding agreement* under  $\tilde{P}$  and  $D(\tilde{P})$  if for any coalition  $\tilde{S}^m \in \tilde{P}$  there is no deviating coalition  $S \subset S^m$  with any distribution rule  $d(S)$ , i.e. there is no pair  $(S, z)$  of a subset  $S \subset S^m$  and a vector  $z = (z_i)_{i \in S} \in R^{|S|}$  of payoffs such that  $\sum_{i \in S} z_i \leq v(S)$  and  $z_i > x_i$  for all  $i \in S$ , where  $z$  is determined by  $d(S)$  and  $x = (x_i)_{i \in N'} \in R^{n+1}$  is determined by the list  $D(\tilde{P})$  of distribution rules under  $\tilde{P}$ .

Given a coalition structure  $\tilde{P}$ , players in each coalition negotiate on how to cooperate to maximize their joint profit and how to split it in a “closed room” in which they cannot communicate with the others outside, but once a coalition  $S$  breaks off the original coalition structure  $\tilde{P}$ , anyone can go outside of the room and talk with the others except members in  $S$  in order to form another coalition structure.

---

<sup>4</sup>Assuming that each coalition  $S$  (without subcoalitions) divides its profit equally among firms in  $S$ , Ray and Vohra (1997) considered an equilibrium binding agreements in a symmetric Cournot competition in their section 7. Our requirements (1), (2) and (3) on distribution rules are jointly equivalent to their assumption.

Under a coalition structure  $\tilde{P}$  with a list  $D(\tilde{P})$  of distribution rules, we regard a strategy profile as the one at least “non-incredible”, if it satisfies the best response property under  $\tilde{P}$ , and if any internal deviation in each coalition is not profitable for members of the deviating coalition  $S$ .

We additionally assume (4)  $d(N', [N']) = \text{Nu}(v)$ , where the *nucleolus*  $\text{Nu}(v)$  satisfies the above three requirements, and is uniquely determined almost in the center of the (shadow) core if it is nonempty (empty). The Shapley value is not necessarily in the core. Hence, the nucleolus is the most appropriate distribution rule under the grand coalition  $N'$ . The definition of nucleolus is described in Appendix.

### 3 Results

Given a coalition structure  $\tilde{P} = [\tilde{S}^0, \dots, \tilde{S}^M]$ , each subcoalition  $S^{l(m)} \in \tilde{S}^m$  maximizes  $\sum_{i \in S^{l(m)}} q_i(p)(p_i - C_i)$ , where  $C_i = c - \epsilon$  if  $\tilde{S}^m = \tilde{S}^0$  and  $C_i = c$  otherwise. Any price such that  $p_i < C_i$  is dominated. Denote by  $s = |S^{l(m)} \setminus \{0\}|$  the number of firms in subcoalition  $S^{l(m)}$ . Any firms  $i$  and  $j$  can be renamed as  $j$  and  $i$ , where  $i, j \in N = \{1, 2, 3\}$ , since they are identical. Hereafter,  $a = a(3) = \alpha d$ ,  $d = d(3)$  and  $e = e(3)$  for short. The first order condition with respect to  $p_i$  is then  $q_i(p) - d(p_i - C_i) + (s - 1)e(p_j - C_i) = 0$ , where  $i, j \in S^{l(m)}$  and  $j \neq i$ . It is easy to show that  $\bar{p}_i = \bar{p}_j$  for any  $i$  and  $j$  in  $S^{l(m)}$ . Hence, we have the following lemma.

**Lemma 1** *When  $\bar{p}_i$  is characterized by the first order condition, we have  $q_i(\bar{p}) = (d - (s - 1)e)(\bar{p}_i - C_i)$ , where  $s = |S^{l(m)} \setminus \{0\}|$ , and so the equilibrium joint profit of subcoalition  $S^{l(m)}$  is given by*

$$\sum_{i \in S^{l(m)}} q_i(\bar{p})(\bar{p}_i - C_i) = s(d - (s - 1)e)(\bar{p}_i - C_i)^2$$

Note that  $d - (s - 1)e > 0$  is met for any  $s \leq 3 = n$  by the assumption that  $0 < \gamma < 1/4 = 1/(n + 1)$ .

We next show Observations for our better understanding the features of equilibrium prices. Proofs are gathered into the proof of Lemma 2, which is in Appendix. Recall that the price index effect is significantly smaller than the own price effect under the assumption  $\gamma < 1/4$ .

**Observation 1** Suppose that there is only one licensee in the industry. Denote by  $p^* = (p_1^*, p_2^*, p_3^*)$ , where  $p_2^* = p_3^*$ , the vector of equilibrium prices under a coalition structure  $\tilde{P}_1 = [\{0\} \cup \{1\}, \{2, 3\}]$  and by  $p' = (p'_1, p'_2, p'_3)$ , where  $p'_2 = p'_3$ , the vector of equilibrium prices under another coalition structure  $\tilde{P}' = [\{0\} \cup \{1\}, \tilde{\pi}']$  with  $\tilde{\pi}' = \{\{2\}, \{3\}\}$  or  $\{2\}, \{3\}$ . Then we have  $p_1^* < p'_1$  and  $p_i^* > p'_i$  for  $i = 2, 3$ .

If non-licensee firms 2 and 3 are not united into one coalition  $\{2, 3\}$  in opposition to the technological advantage of licensee firm 1, their mutual antagonistic pricing lower their own equilibrium prices  $p_2$  and  $p_3$ , whereas it is more profitable for the licensee firm 1 to set a higher price  $p'_1 (> p_1^*)$ , because of the small decrement in demand for firm 1's commodity through the price index effect. ( $p_1^* > p'_1$  and  $p_i^* > p'_i$  for  $i = 2, 3$  if  $\gamma \gg 1/4$ .)

**Observation 2** Suppose that there is two licensees in the industry. Denote by  $p^{**} = (p_1^{**}, p_2^{**}, p_3^{**})$ , where  $p_1^{**} = p_2^{**}$ , the vector of equilibrium prices under a coalition structure  $\tilde{P}_2 = [\{0\} \cup \{1, 2\}, \{3\}]$  and by  $p'' = (p''_1, p''_2, p''_3)$ , where  $p''_1 = p''_2$ , the vector of equilibrium prices under another coalition structure  $\tilde{P}'' = [\pi'', \{3\}]$  with  $\pi'' \setminus \{0\} = \{\{1\}, \{2\}\}$ . Then we have  $p_i^{**} > p''_i$  for  $i = 1, 2$  and  $p_3^{**} < p''_3$ .

The competition between licensee firms 1 and 2 does not result in the effective use of their technological advantage. Their mutual antagonistic pricing lower their own equilibrium prices  $p_1$  and  $p_2$ , whereas it leads a higher equilibrium price  $p''_3 (> p_3^{**})$  of non-licensee firm 3 through the small price index effect on the demand for firm 3's commodity. ( $p_i^{**} > p''_i$  for  $i = 1, 2$  and  $p_3^{**} < p''_3$  if  $\gamma \gg 1/4$ .)

The following v-function reflects the features described in Observations.

**Lemma 2**

$$v(\{0\}) = v(\emptyset) = 0. \quad (1)$$

$$v(\{0\} \cup \{i\}) = d(p_i^* - c + \epsilon)^2, \quad (2)$$

where  $p_i^* = ((a + cd)d - ce^2 - \epsilon d(d - e))/(2d^2 - e^2 - 2de)$  is obtained under coalition structure  $\tilde{P}_1 = [\{0\} \cup \{i\}, \{j, k\}]$ .

$$v(\{0\} \cup \{i, j\}) = 2(d - e)(p_i^{**} - c + \epsilon)^2, \quad (3)$$

where  $p_i^{**} = p_j^{**} = (2(a + cd)d + (a - cd)e - 2\epsilon d(d - e))/2(2d^2 - e^2 - 2de)$  is obtained under coalition structure  $\tilde{P}_2 = [\{0\} \cup \{i, j\}, \{k\}]$ .

$$v(N') = v(\{0\} \cup \{i, j, k\}) = 3(d - 2e)(\bar{p}_i^{**} - c + \epsilon)^2, \quad (4)$$

where  $\bar{p}_i^{**} = \bar{p}_j^{**} = \bar{p}_k^{**} = (a + (c - \epsilon)(d - 2e))/2(d - 2e)$  is obtained under a coalition structure  $\tilde{P}_3 = [\{0\} \cup \{i, j, k\}]$ .

$$v(\{k\}) = d(p_k^{**} - c)^2, \quad (5)$$

where  $p_k^{**} = ((a + cd)d - ce^2 - \epsilon e(d - e))/(2d^2 - e^2 - 2de)$  is obtained under coalition structure  $\tilde{P}_2 = [\{0\} \cup \{i, j\}, \{k\}]$ .

$$v(\{j, k\}) = 2(d - e)(p_j^* - c)^2, \quad (6)$$

where  $p_j^* = p_k^* = (2(a + cd)d + (a - cd)e - \epsilon de)/(2(2d^2 - e^2 - 2de))$  is obtained under coalition structure  $\tilde{P}_1 = [\{0\} \cup \{i\}, \{j, k\}]$ .

$$v(N) = v(\{i, j, k\}) = 3(d - 2e)(\bar{p}_i^* - c)^2, \quad (7)$$

where  $\bar{p}_i^* = \bar{p}_j^* = \bar{p}_k^* = (a + c(d - 2e))/2(d - 2e)$  is obtained under coalition structure  $\tilde{P}_0 = [\{0\}, \{i, j, k\}]$ .

*Proof*: See Appendix.

**Lemma 3**  $\text{Nu}_0(v) = (v(N') - v(N))/2$  is the nucleolus of the patent holder.

*Proof*: See Appendix.

**Proposition 1** (1) Under  $\tilde{P}_1 = [\{0\} \cup \{i\}, \{j, k\}]$ , there are distribution rules under which the strategy profile  $p^* = (p_i^*, p_j^*, p_k^*)$  is a binding agreement.

(2) Under  $\tilde{P}_2 = [\{0\} \cup \{i, j\}, \{k\}]$ , there is a real number  $0 < \gamma_2(\alpha, c, \epsilon) < 1/7$  such that there are distribution rules under which the strategy profile  $p^{**} = (p_i^{**}, p_j^{**}, p_k^{**})$  is a binding agreement if and only if  $\gamma \leq \gamma_2(\alpha, c, \epsilon)$ , and such that  $\partial\gamma_2(\alpha, c, \epsilon)/\partial\epsilon > 0$ .

(3) Under  $\tilde{P}_3 = [\{0\} \cup \{i, j, k\}]$ , there are distribution rules under which the strategy profile  $\bar{p}^{**} = (\bar{p}_i^{**}, \bar{p}_j^{**}, \bar{p}_k^{**})$  is a binding agreement.

*Proof*: See Appendix.

It is now questioned which coalition structure gives the highest payoff to the patent holder. (Proposition 2 does not refer to binding agreements.)

**Proposition 2** (1) Under  $\tilde{P}_2$ , there are distribution rules  $d(\{0\} \cup \{i, j\}, \tilde{P}_2)$  which give the patent holder more amount of payoff than  $d(N', \tilde{P}_3) = \text{Nu}(v)$  and any  $d(\{0\} \cup \{i\}, \tilde{P}_1)$ .

(2) Under  $\tilde{P}_1$ , there are distribution rules  $d(\{0\} \cup \{i\}, \tilde{P}_1)$  which give the patent holder more amount of payoff than  $d(N', \tilde{P}_3) = \text{Nu}(v)$  if  $\gamma_1(\alpha, c, \epsilon) < \gamma$ , where  $1/7 < \gamma_1(\alpha, c, \epsilon) < 1/6$ .

*Proof*: See Appendix.

Proposition 1 and 2 jointly implies that if the patent holder can choose firms to negotiate with on the licensing issues and (binding) agreements, it is then most profitable for him or her to license its new technology to two firms when the commodities are not so substitutive ( $\gamma < \gamma_2(\alpha, c, \epsilon)$ ), or to only one firm when they are quite substitutive ( $\gamma_1(\alpha, c, \epsilon) < \gamma$ ), or to all the three firms when they are moderate ( $\gamma_2(\alpha, c, \epsilon) < \gamma < \gamma_1(\alpha, c, \epsilon)$ ).

## 4 Discussion

Recall Observations. Against the collusive lower pricing by two firms, it is optimal for the other firm to raise the unit price while losing some amount of the market demand. That is due to the low magnitude of substitutions among commodities, i.e.  $\gamma < 1/4$ . If  $\gamma$  is close to 1, it would then be better for the firm to lower the price in opposition to such a collusive lower pricing. Higher magnitudes of substitutions lead the higher worth of cooperation in the grand coalition  $N$ , and then it would be optimal for the patent holder to license its new technology to all the three firms.

We can find a similar analysis also in Zhao (1999). He showed sufficient conditions for the existence of *hybrid solution with distribution rules* (HSDR) for a coalition structure  $P$  (a partition of players in this paper) with a list  $D(P) = (d(S^0, P), \dots, d(S^M, P))$  of distribution rules, where  $d(S^m, P)$  is  $\beta$ -core for every coalition (without subcoalitions)  $S^m \in P$ . Given a list  $D(\tilde{P}) = (d(\tilde{S}^0, \tilde{P}), \dots, d(\tilde{S}^M, \tilde{P}))$  of distribution rules, a strategy profile  $\bar{p} \in \beta(\tilde{P})$  that satisfies the best response property in this paper is actually the same as a HSDR, except spin-offs of subcoalitions.

He also defined the stability of a HSDR for  $P$ , specifying possible new coalition structures that form after a deviating coalition  $S$  breaks off the original one. The binding agreement in this chapter corresponds to such a stability. The distribution rules such as nucleolus, kernel, bargaining set need a  $v$ -function, since they are defined with it. In his definition, however,  $v$ -function is derived in the classical way, which would be inconsistent with the specification of new coalition structures posterior to a deviation.

At last, it would not be difficult to extend this triopoly to  $n$ -firm model, if we could start with demand functions, not with the utility function of the representative consumer. We have to simplify the relationship among the parameters to go further. The existence result of a binding agreement in general strategic-form games should be similar to Zhao's.

## Appendix

### The Nucleolus

Denote by  $w = (w_0, w_1, \dots, w_n) \in \mathbb{R}^{n+1}$  a vector of payoffs, where  $w_0$  is the patent holder's payoff and  $w_j$  is firm  $j$ 's. The *imputation* of a game  $(N', v)$  is defined as  $I(v) = \{w \in \mathbb{R}^{n+1} : \sum_{i \in N'} w_i = v(N') \text{ and } w_i \geq v(\{i\})\}$ .

For each payoff vector  $w$ , denote by  $e(S, w) = v(S) - \sum_{i \in S} w_i$  the *excess* of coalition  $S$  to  $w$ . It measures the amount of sacrifice (surplus) of  $S$ , if positive (negative), when  $w$  is implemented.

Let  $\theta(w) = (e(S_1, w), e(S_2, w), \dots, e(S_{2^n}, w))$  be a vector of excesses of all coalitions except  $N'$  arranged in such an order that  $e(S_t, w) \geq e(S_{t+1}, w)$ .

We say that  $\theta(w)$  is *lexicographically smaller* than  $\theta(w')$  and denote it by  $\theta(w) \preceq \theta(w')$  if there exists a positive integer  $t'$  such that  $\theta_t(w) = \theta_t(w')$  whenever  $1 \leq t < t'$  and  $\theta_{t'}(w) < \theta_{t'}(w')$ .

**Definition 4** The *nucleolus* (Schmeidler (1969)) of  $(N', v)$  is defined as

$$\text{Nu}(v) = \{w \in I(v) : \theta(w) \preceq \theta(w') \text{ for all } w' \in I(v) - \{w\}\}.$$

Let  $x_T = (x_i)_{i \in T}$  be the projection of  $x \in I(v)$  to the subspace that corresponds to the coalition  $T$ . Denote by  $v_{T,x}$  the reduced game of  $(N', v)$ , i.e.,  $v_{T,x}(T) = v(N') - \sum_{i \in N' \setminus T} x_i$ ;  $v_{T,x}(S) = \max_{Q \subseteq N' \setminus T} v(S \cup Q) - \sum_{i \in Q} x_i$  for  $S \subset T$ ;  $v_{T,x}(\{\emptyset\}) = 0$ . In addition to requirements (1), (2) and (3), the nucleolus satisfies the following property.

**Definition 5** the *reduced game property* (Davis and Maschler (1965)) : If  $x = \text{Nu}(v)$ , then  $x_T = \text{Nu}(v_{T,x})$  for all  $T \subset N'$ .

We use this property to show the proof of Lemma 3.

## Proof of Lemma 2

It suffices to show the following cases, since every firm  $i \in N$  is initially identical. Recall that  $a = a(3) = \alpha d$ ,  $d = d(3) = (1 - \gamma)/(2 - \gamma - \gamma^2)$  and  $e = e(3) = \gamma/(2 - \gamma - \gamma^2)$  for short.

(case 1)  $S^0 = \{0\} \cup \{1\}$  and  $N' \setminus S^0 = \{2, 3\}$ .

Let us begin with calculating the guaranteed worths  $v(S^0)$  and  $v(N' \setminus S^0)$  of coalition  $S^0 = \{0\} \cup \{1\}$  and  $N' \setminus S^0 = \{2, 3\}$ . Possible coalition structures over  $N' \setminus S^0$  are  $\tilde{\pi} = [\{2, 3\}]$ ,  $[\{\{2\}, \{3\}\}]$  or  $[\{2\}, \{3\}]$ .

Let  $\tilde{\pi} = \{2, 3\}$ , first. Given a coalition structure  $\tilde{P}_1 = [\{0\} \cup \{1\}, \tilde{\pi}]$ , the first order conditions are  $a + (c - \epsilon)d + ep_2 + ep_3 = 2dp_1$ ,  $a + c(d - e) + ep_1 = 2dp_2 - 2ep_3$  and  $a + c(d - e) + ep_1 = 2dp_3 - 2ep_2$ , respectively. The interior solution  $p^* = (p_1^*, p_2^*, p_3^*)$ , where  $p_3^* = p_2^*$ , under  $\tilde{P}$  is given by

$$(p_1^*, p_2^*) = \left( \frac{(a + cd)d - ce^2 - \epsilon d(d - e)}{2d^2 - e^2 - 2de}, \frac{2(a + cd)d + ae - (c + \epsilon)de}{2(2d^2 - e^2 - 2de)} \right).$$

We have  $\bar{p} = p^*$  if  $p_2^* = p_3^* > c$ , i.e.  $de\epsilon < 2(a - cd)d + 2ce^2 + 3cde + ae = 2(\alpha - c)d^2 + 2ce^2 + (3c + \alpha)de$ . This condition is always met since  $de > 0$  and  $\alpha > c > \epsilon$ .

Consider next the case where  $\tilde{\pi}' = \{\{2\}, \{3\}\}$  or  $\{2\}, \{3\}$ . Let  $\tilde{P}'_1 = [\{0\} \cup \{1\}, \tilde{\pi}']$ . Since firms 2 and 3 are in different subcoalitions, the first order conditions with respect to  $p_2$  and  $p_3$  are the same in both cases, and they are  $a + cd + ep_1 = 2dp_i - ep_j$ , where  $i, j = 2, 3$ . It is  $a + (c - \epsilon)d + ep_2 + ep_3 = 2dp_1$  with respect to  $p_1$ . The interior solution  $p' = (p'_1, p'_2, p'_3)$ , where  $p'_3 = p'_2$ , under  $\tilde{P}'$  is given by

$$(p'_1, p'_2) = \left( \frac{2(a + cd)d + ced + ae - \epsilon d(2d - e)}{2(2d^2 - e^2 - de)}, \frac{2(a + cd)d + ae + (c - \epsilon)de}{2(2d^2 - e^2 - de)} \right),$$

We have  $\bar{p} = p'$ , since  $p'_3 = p'_2 > c$ .

It is confirmed that  $p_1^* < p_1'$  and  $p_2^* > p_2'$ . Lemma 1 shows that the equilibrium profit of subcoalitions  $S^{l(m)}$  is  $s(d - (s - 1)e)(\bar{p}_i - C_i)^2$ , where  $s = |S^{l(m)} \setminus \{0\}|$ . Since  $N' \setminus S^0 = \{2, 3\}$  takes the form of a coalition structure over  $N' \setminus S^0$  as  $\{\{2\}, \{3\}\}$  or  $\{2\}, \{3\}$  so as to minimize the equilibrium joint profit of  $S^0$ , we have  $v(\{0\} \cup \{1\}) = d(p_1^* - c + \epsilon)^2$  by  $p_1^* < p_1'$ .

We next show that  $v(\{2, 3\}) = 2(d - e)(p_2^* - c)^2$ . Compare the equilibrium joint profit of  $S = \{2, 3\}$  between the two cases  $\pi = \{2, 3\}$  and  $\pi' = \{\{2\}, \{3\}\}$ , i.e.  $2(d - e)(p_2^* - c)^2$  and  $2d(p_2' - c)^2$  by Lemma 1. In the range of  $0 < \gamma < 1/4$ ,  $p_2^* - p_2' (> 0)$  is increasing in  $\gamma$ .  $e = \gamma/(1 - \gamma + 1 - \gamma^2)$  is also increasing in  $\gamma$ . Hence, we have  $2(d - e)(p_2^* - c)^2 > 2d(p_2' - c)^2$ , if it holds as  $\gamma \rightarrow 1/4$ . As  $\gamma \rightarrow 1/4$ , we have

$$\begin{aligned} 2(d - e)(p_2^* - c)^2 &\rightarrow (16/27)\left(\frac{21\alpha - 7c - 3\epsilon}{22}\right)^2 \\ 2d(p_2' - c + \epsilon)^2 &\rightarrow (8/9)\left(\frac{21\alpha - 7c - 3\epsilon}{28}\right)^2. \end{aligned}$$

The former is bigger than the latter. Hence,  $2(d - e)(p_2^* - c)^2 > 2d(p_2' - c)^2$  for any  $\gamma \in (0, 1/4)$ . Since  $S = \{2, 3\}$  takes the form of subcoalitions as  $\{2, 3\}$  so as to maximize the equilibrium joint profit of  $S$ , we have  $v(\{2, 3\}) = 2(d - e)(p_2^* - c)^2$ .

(case 2)  $S^0 = \{0\} \cup \{1, 2\}$  and  $N' \setminus S^0 = \{3\}$ .

Let us calculate the guaranteed worths  $v(S^0)$  and  $v(N' \setminus S^0)$  of coalition  $S^0 = \{0\} \cup \{1, 2\}$  and  $N' \setminus S^0 = \{3\}$ . Possible subcoalitions  $\pi \setminus \{0\}$  of firms is  $\{1, 2\}$  or  $\{\{1\}, \{2\}\}$ .

Let  $\pi \setminus \{0\} = \{1, 2\}$ , first. Given a coalition structure  $\tilde{P}_2 = [\pi, \{3\}]$ , the first order conditions are  $a + (c - \epsilon)(d - e) + ep_3 = 2dp_1 - 2ep_2$ ,  $a + (c - \epsilon)(d - e) + ep_3 = 2dp_2 - 2ep_1$ , and  $a + cd + ep_1 + ep_2 = 2dp_3$ , respectively. The interior solution  $p^* = (p_1^*, p_2^*, p_3^*)$ , where  $p_2^* = p_1^*$ , under  $\tilde{P}$  is given by

$$(p_1^*, p_3^*) = \left( \frac{2(a + cd)d + (a - cd)e - 2\epsilon d(d - e)}{2(2d^2 - e^2 - 2de)}, \frac{ad + c(d^2 - e^2) - \epsilon e(d - e)}{2d^2 - e^2 - 2de} \right).$$

We have  $\bar{p} = p^*$  if  $p_3^* > c$ , i.e.  $e(d - e)\epsilon < d(a - cd + 2ce)$ . This condition is always met since  $0 < \gamma < 1/4$  and  $\alpha > c > \epsilon$ .

Consider next the case where  $\pi'' \setminus \{0\} = \{\{1\}, \{2\}\}$ . Let  $\tilde{P}'_2 = [\pi'', \{3\}]$ . The first order conditions with respect to  $p_i$  is  $a + d(c - \epsilon) + ep_3 = 2dp_i - ep_j$ , where  $i, j = 1, 2$ . It is  $a + cd + ep_1 + ep_2 = 2dp_3$  with respect to  $p_3$ . The interior solution  $p'' = (p''_1, p''_2, p''_3)$ , where  $p''_2 = p''_1$ , under  $\tilde{P}'$  is given by

$$(p''_1, p''_3) = \left( \frac{(e + 2d)(a + cd) - 2e\epsilon d^2}{2(2d^2 - e^2 - de)}, \frac{(e + 2d)(a + cd) - 2e\epsilon d}{2(2d^2 - e^2 - de)} \right),$$

We have  $\bar{p} = p''$ , since  $p''_3 > c$ .

It is confirmed that  $p_1^* > p''_1$  and  $p_3^* < p''_3$ . Lemma 1 shows that the equilibrium profits of subcoalitions  $S^{l(m)}$  is  $s(d - (s - 1)e)(\bar{p}_i - C_i)^2$ , where  $s = |S^{l(m)} \setminus \{0\}|$ . Since  $S^0 \setminus \{0\}$  takes the form of a coalition structure of firms over  $S^0 \setminus \{0\}$  as  $\{1, 2\}$  so as to minimize the equilibrium joint profit of  $N' \setminus S^0 = \{3\}$ , we have  $v(\{3\}) = d(p_3^* - c)^2$  by  $p_3^* < p''_3$ .

We next show that  $v(\{0\} \cup \{1, 2\}) = 2(d - e)(p_1^* - c)^2$ . Compare the equilibrium joint profit of  $S^0 = \{0\} \cup \{1, 2\}$  between the two cases  $\pi \setminus \{0\} = \{1, 2\}$  and  $\pi'' \setminus \{0\} = \{\{1\}, \{2\}\}$ , i.e.  $2(d - e)(p_1^* - c + \epsilon)^2$  and  $2d(p''_1 - c + \epsilon)^2$  by Lemma 1. In the range of  $0 < \gamma < 1/4$ ,  $p_1^* - p''_1 (> 0)$  is increasing in  $\gamma$ .  $e$  is also increasing in  $\gamma$ , Hence, we have  $2(d - e)(p_1^* - c + \epsilon)^2 > 2d(p''_1 - c + \epsilon)^2$  if it holds as  $\gamma \rightarrow 1/4$ . As  $\gamma \rightarrow 1/4$ , we have

$$\begin{aligned} 2(d - e)(p_1^* - c + \epsilon)^2 &\rightarrow (16/27) \left( \frac{21\alpha - 7c + 10\epsilon}{22} \right)^2 \\ 2d(p''_1 - c + \epsilon)^2 &\rightarrow (8/9) \left( \frac{21\alpha - 7c + 10\epsilon}{28} \right)^2. \end{aligned}$$

The former is bigger than the latter. Hence, we have  $2(d - e)(p_1^* - c + \epsilon)^2 > 2d(p''_1 - c + \epsilon)^2$  for any  $\gamma \in (0, 1/4)$ . Since  $S^0 \setminus \{0\} = \{1, 2\}$  takes the form of subcoalitions of firms as  $\{1, 2\}$  so as to maximize the equilibrium joint profit of  $S^0$ , we have  $v(\{0\} \cup \{1, 2\}) = 2(d - e)(p_1^* - c + \epsilon)^2$ .

(case 3)  $S = \{1, 2, 3\} = N$ .

Let us calculate the guaranteed worth  $v(N)$  of coalition  $N$ . Possible subcoalitions are  $\pi_N = \{1, 2, 3\}$ ,  $\pi_{12} = \{\{1, 2\}, \{3\}\}$  or  $\pi_1 = \{\{1\}, \{2\}, \{3\}\}$ .

When  $\pi_1 = \{\{1\}, \{2\}, \{3\}\}$ , we have  $\bar{p}_i = (a + cd)/2(d - e)$  for any  $i = 1, 2, 3$  by the symmetric first order conditions. Similarly, we have

$$\bar{p}_i^* = \frac{a + c(d - 2e)}{2(d - 2e)}$$

for any  $i = 1, 2, 3$ , when  $\pi_N = \{1, 2, 3\}$ . By Lemma 1, the equilibrium joint profit of  $N$  are  $3d(\bar{p}_i - c)^2$  and  $3(d - 2e)(\bar{p}_i^* - c)^2$ , respectively. It is confirmed that  $3d(\bar{p}_i - c)^2 < 3(d - 2e)(\bar{p}_i^* - c)^2$ .

The remaining case is  $\pi_{12} = \{\{1, 2\}, \{3\}\}$ . By the first order conditions,  $\bar{p}^\dagger = (\bar{p}_1^\dagger, \bar{p}_2^\dagger, \bar{p}_3^\dagger)$ , where  $\bar{p}_2^\dagger = \bar{p}_1^\dagger$ , under  $\tilde{P} = [\{0\}, \pi_{12}]$  is given by

$$(\bar{p}_1^\dagger, \bar{p}_3^\dagger) = \left( \frac{ae - cde + 2ad + 2cd^2}{2(2d^2 - e^2 - 2de)}, \frac{ad + cd^2 - ce^2}{2d^2 - e^2 - 2de} \right).$$

By Lemma 1, the equilibrium joint profit is  $2(d - e)(\bar{p}_1^\dagger - c)^2 + d(\bar{p}_3^\dagger - c)^2$ .

Compare the equilibrium joint profit of  $N$  between the two cases  $\pi_N = \{1, 2, 3\}$  and  $\pi_{12} = \{\{1, 2\}, \{3\}\}$ . In the range of  $0 < g < 1/4$ ,  $\bar{p}_i^* - \bar{p}_i^\dagger (> 0)$ , where  $i = 1, 2, 3$ , is increasing in  $\gamma$ .  $e$  is also increasing in  $\gamma$ . Hence, we have  $3(d - 2e)(\bar{p}_i^* - c)^2 > 2(d - e)(\bar{p}_1^\dagger - c)^2 + d(\bar{p}_3^\dagger - c)^2$  if it holds as  $\gamma \rightarrow 1/4$ . As  $\gamma \rightarrow 1/4$ , we have

$$\begin{aligned} 3(d - 2e)(\bar{p}_i^* - c)^2 &\rightarrow (1/9)(3\alpha - c)^2 \\ 2(d - e)(\bar{p}_1^\dagger - c)^2 + d(\bar{p}_3^\dagger - c)^2 &\rightarrow (15/27)(7/22)^2(3\alpha - c)^2 \\ &\quad + (4/121)(3\alpha - c)^2. \end{aligned}$$

The former is bigger than the latter. Hence, we have  $3(d - 2e)(\bar{p}_i^* - c)^2 > 2(d - e)(\bar{p}_1^\dagger - c)^2 + d(\bar{p}_3^\dagger - c)^2$  for any  $\gamma \in (0, 1/4)$ . Since  $N$  takes the form of subcoalitions as  $\pi_N = \{1, 2, 3\}$  so as to maximize the equilibrium joint profit of  $N$ , we have  $v(N) = 3(d - 2e)(\bar{p}_i^* - c)^2$ .

(case 4)  $S^0 = \{0\} \cup \{1, 2, 3\} = N'$ .

Replace  $c$  in case 3 with  $c - \epsilon$ . Then we have  $v(N') = 3(d - 2e)(\bar{p}_i^{**} - c + \epsilon)^2$ , where  $\bar{p}_i^{**} = (a + (c - \epsilon)(d - 2e))/(2(d - 2e))$ . *Q.E.D.*

### Proof of Lemma 3

Denote by  $\text{Nu}(v) = x = (x_0, x_1, x_2, x_3)$  the nucleolus of  $(N', v)$ , where  $x_0$  is the patent holder's and  $x_j$  is firm  $j$ 's. First, consider a bargaining between the patent holder and the "union"  $N$  of all the firms. Then, a vector  $y = (y_0, y_1, y_2, y_3)$  of payoffs, where  $y_0 = v(\{0\}) + (v(N') - v(\{0\}) - v(N))/2$  and  $y_i = (1/3)(v(N) + (v(N') - v(\{0\}) - v(N))/2)$  for all  $i \in N$  equalizes the excesses of  $\{0\}$  and  $N$  as well as those among firms. Hence,  $y$  is a candidate for  $\text{Nu}(v)$ .

Let  $T = \{0, i\}$ . In the range of  $0 < \gamma < 1/4$ ,  $3v(\{0\} \cup \{i\}) < v(N)$  and  $3v(\{0\} \cup \{i, j\}) < 2v(N)$ , and so  $v(\{0\} \cup \{i\}) - y_i < 0$  and  $v(\{0\} \cup \{i, j\}) - (y_i + y_j) < 0$ . Similarly,  $v_{T,y}(\{i\}) = \max_{Q \subseteq \{j,k\}} v(\{i\} \cup Q) - \sum_{i \in Q} y_i = v(N) - (y_j + y_k)$ . Hence, the reduced game  $v_{T,x}$  of  $(N', v)$  is as follows.

$$\begin{aligned} v_{T,y}(T) &= v(N') - (y_j + y_k) \\ v_{T,y}(\{1\}) &= v(N) - (y_j + y_k) \\ v_{T,y}(\{0\}) &= v(\{0\}) = 0. \end{aligned}$$

Equalizing the excesses of  $\{0\}$  and  $\{i\}$ , we have

$$\text{Nu}_0(v_{T,y}) = v_{T,y}(\{0\}) + \frac{v_{T,y}(T) - v_{T,y}(\{0\}) - v_{T,x}(\{1\})}{2} = \frac{v(N') - v(N)}{2}.$$

Hence,  $y$  satisfies the reduced game property.

Other candidates  $z$  for  $\text{Nu}(v)$  are such that  $z_i = z_j = z_k \geq v(\{k\})$  and  $z_0 = v(N') - 3z_i \geq v(\{0\}) = 0$ , but it is easy to see that any  $z \neq y$  does not satisfy the reduced game property, since  $3v(\{0\} \cup \{i\}) < v(N)$  and  $3v(\{0\} \cup \{i, j\}) < 2v(N)$  in the range of  $0 < \gamma < 1/4$ . *Q.E.D.*

### Proof of Proposition 1

(1) Lemma 2 shows that the worth  $v(S^0)$  of coalition  $S^0 = \{0\} \cup \{i\}$  is the equilibrium joint profit under a coalition structure  $\tilde{P}_1 = [\{0\} \cup \{i\}, \{j, k\}]$ . Since  $v(\{0\} \cup \{i\}) > v(\{i\}) > v(\{0\}) = 0$ , for example, the strategy profile  $p^* = (p_i^*, p_j^*, p_k^*)$  is a binding agreement under distribution rules  $d(S^0, \tilde{P}_1) = (x_0 = (v(\{0\} \cup \{i\}) - v(\{i\}))/2, x_i = v(\{0\} \cup \{i\}) - x_0)$  and  $d(\{j, k\}, \tilde{P}_1) = (v(\{j, k\})/2, v(\{j, k\})/2)$ .

Now, let  $\tilde{P}'_1 = [\{0\} \cup \{i\}, \{\{j\}, \{k\}\}]$ . As shown in case 1 in the Proof of Lemma 2,  $S = \{j, k\}$  takes the form of subcoalitions as  $\{j, k\}$  so as to maximize the equilibrium joint profit of  $S$ . Hence, any vector of prices under  $\tilde{P}' = [\{0\} \cup \{i\}, \{\{j\}, \{k\}\}]$  cannot be a binding agreement.

(2) Let us next consider the case of  $\tilde{P}_2 = [\{0\} \cup \{i, j\}, \{k\}]$ . There exists a distribution rule under which the strategy profile  $p^{**} = (p_i^{**}, p_j^{**}, p_k^{**})$  is a binding agreement, if the following system of inequalities holds.

$$\begin{aligned} v(\{0\} \cup \{i, j\}) - v(\{i, j\}) &\geq x_0 \geq 0 \\ v(\{0\} \cup \{i, j\}) - v(\{0\} \cup \{i\}) &\geq x_j \geq v(\{j\}) = v(\{k\}) \\ v(\{0\} \cup \{i, j\}) &\geq x_i + x_j \geq v(\{i, j\}). \end{aligned}$$

The first and last lines hold obviously. By Lemma 2,

$$\begin{aligned} v(\{0\} \cup \{i, j\}) &= 2(d - e)(p_i^{**} - c + \epsilon)^2 \\ v(\{0\} \cup \{i\}) &= d(p_i^* - c + \epsilon)^2 \\ v(\{k\}) &= d(p_k^{**} - c), \end{aligned}$$

where  $p_i^{**} > p_i^* > p_k^{**}$  by  $0 < \gamma < 1/4$ . The second line then holds as  $\gamma \rightarrow 0$ . If  $\gamma$  is far from 0, the second line does not necessarily hold, since  $\lim_{\gamma \rightarrow 1/4} \{2(d - e)(p_i^{**} - c + \epsilon)^2 - d(p_i^* - c + \epsilon)^2 - d(p_k^{**} - c)\} = -(20/363)\alpha^2 + (40/1089)\alpha c + (344/1089)\alpha \epsilon - (20/3267)c^2 - (344/3267)c\epsilon + (52/3267)\epsilon^2$  is not necessarily positive. In the range of  $0 < \gamma < 1/4$ ,  $p_i^{**} - p_i^* - p_k^{**}$  is decreasing in  $\gamma$ .  $e = \gamma/(2 - \gamma - \gamma^2)$  is increasing in  $\gamma$ . Hence, there should be a real number  $\gamma_2(\gamma, \epsilon) > 0$  such that there are distribution rules under

which the strategy profile  $p^{**} = (p_i^{**}, p_j^{**}, p_k^{**})$  is a binding agreement if and only if  $\gamma \leq \gamma_2(\gamma, \epsilon)$ . Since  $\alpha > c > \epsilon$ , it is confirmed by direct calculations that  $\gamma_2(\gamma, \epsilon) \in (0, 1/7)$  and that  $\partial\gamma_2(\gamma, \epsilon)/\partial\epsilon > 0$ .

By Lemma 2, we can confirm that any vector of prices under  $\tilde{P}'_2 \neq \tilde{P}_2$  cannot be a binding agreement.

(3) Consider the case of  $\tilde{P}_3 = [\{0\} \cup \{i, j, k\}]$ . There are distribution rules under which the strategy profile  $\bar{p}^{**} = (\bar{p}_i^{**}, \bar{p}_j^{**}, \bar{p}_k^{**})$  is a binding agreement, if the following system of inequalities holds with  $\delta \leq 0$ .

$$\begin{aligned} v(N') - v(N) + \delta &\geq x_0 &\geq -\delta \\ v(N') - v(\{0\} \cup \{i, j\}) + \delta &\geq x_k &\geq v(\{k\}) - \delta \\ v(N') - v(\{0\} \cup \{i\}) + \delta &\geq x_j + x_k &\geq v(\{j, k\}) - \delta \\ v(N') + \delta &\geq x_i + x_j + x_k &\geq v(N) - \delta. \end{aligned}$$

In fact, it holds even with  $\delta = -(v(N') - v(N))/2$ , since  $\text{Nu}(v)$  is obtained with such  $\delta$ . *Q.E.D.*

## Proof of Proposition 2

(1) Let  $\tilde{P}_1 = [\{0\} \cup \{i\}, \{j, k\}]$ ,  $\tilde{P}_2 = [\{0\} \cup \{i, j\}, \{k\}]$  and  $\tilde{P}_3 = [N']$ . For a coalition structure  $\tilde{P} = \tilde{P}_1$  or  $\tilde{P}_2$ , a distribution rule  $d(\tilde{S}^0, \tilde{P})$  that gives  $v(S^0) - v(S^0 \setminus \{0\})$  to the patent holder and  $v(S^0 \setminus \{0\})/|S^0 \setminus \{0\}|$  to each firm in  $S^0$  satisfies all the requirements on distribution rules.

By assumptions that  $\alpha > c > \epsilon$  and that  $0 < \gamma < 1/4$ , we have that  $v(\{0\} \cup \{i, j\}) - v(\{i, j\}) > v(\{0\} \cup \{i\}) - v(\{i\})$  and  $v(\{0\} \cup \{i, j\}) - v(\{i, j\}) > \text{Nu}_0(v) = (v(N') - v(N))/2$ . This implies that there are distribution rules  $d(\{0\} \cup \{i, j\}, \tilde{P}_2)$  that gives the patent holder more amount of payoff than  $d(N', \tilde{P}_3) = \text{Nu}(v)$  and any  $d(\{0\} \cup \{i\}, \tilde{P}_1)$ , since  $v(\{0\} \cup \{i\}) - v(\{i\})$  is the highest payoff for the patent holder under  $\tilde{P}_1$ .

(2) Recall that  $\alpha > c > \epsilon$ . Direct calculations with  $\gamma \leq 1/7$  and  $\gamma \geq 1/6$  show the result. *Q.E.D.*

## References

- [1] Aumann, R. J. (1959) "Acceptable Points in General Cooperative  $n$ -Person Games," in *Contributions to the Theory of Games IV*, H. W. Kuhn and R. D. Luce eds. Princeton University Press, 287-324
- [2] Davis, M., and M. Maschler (1965) "The Kernel of a Cooperative Game," *Naval Research Logistics Quarterly* 223-259
- [3] Kamien, M. I., S. S. Oren and Y. Tauman (1992) "Optimal Licensing of Cost-reducing Innovation," *Journal of Mathematical Economics* 21, 483-508
- [4] Kamien, M. I. and Y. Tauman (1986) "Fees versus Royalties and the Private Value of a Patent," *Quarterly Journal of Economics* 101, 471-491
- [5] Maschler, M., B. Peleg and L. Shapley (1979) "Geometric Properties of the Kernel, Nucleolus and Related Solution Concepts," *Mathematics of Operations Research* 4, 303-338
- [6] Muto, S. (1993) "On Licensing Policies in Bertrand Competition," *Games and Economic Behavior* 5, 257-267
- [7] Ray, D. and R. Vohra (1997) "Equilibrium Binding Agreements," *Journal of Economic Theory* 73, 30-78
- [8] Schmeidler, D. (1969) "The Nucleolus of a Characteristic Function Game," *SIAM Journal of Applied Mathematics* 17, 1163-1170
- [9] Singh, N. and X. Vives (1984) "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics* 15, 546-554
- [10] von Neumann, J and O. Morgenstern (1944) *Theory of Games and Economic Behavior*, Princeton University Press, 2nd ed. 1947

- [11] Watanabe, N. and Y. Tauman (2003) "Asymptotic Properties of the Shapley Value of Patent Licensing Games," working paper, Department of Economics, SUNY at Stony Brook
- [12] Zhao, J. (1999) "A Beta Core Existence Result and Its Application to Oligopoly Market," *Games and Economic Behavior* 27, 153-168